Bridgetown Primary School Calculation Policy

This document shows progression in calculation (addition, subtraction, multiplication and division) in line with the National Curriculum. The consistent use of the CPA (concrete, pictorial, abstract) approach within our school helps pupils develop mastery across all the operations in an efficient and reliable way. Our calculation policy supports staff and parents by demonstrating how these methods progress to develop pupils' confidence, proficiency and deepen their understanding of both written and mental methods.

Sound mental fluency underpins written calculations, enabling pupils to make choices about when a written method should be used and when a mental strategy would be most appropriate. Correct modelling of mathematical language is key to helping pupils explain their methods and develop links between mathematical operations.

Glossary

Correct modelling of mathematical vocabulary is essential in order for pupils to explain and understand mathematical structures. The following terms should be introduced as pupils progress throughout the school. Just as we have technical terms for phonics, e.g. diagraph, trigraph, which we teach from a young age, we should not shy away from using the appropriate vocabulary to describe numbers in mathematical contexts e.g. addend, subtrahend.

Addend A number to be added to another number. e.g. 7+8= 15 7 and 8 are addends	**Aggregation** To combine 2 or more quantities or measure to find a total. e.g. Mina has 3 pencils and George has 2, how many pencils are there altogether?	**Array** An ordered collection of counters, cubes or other items in rows and columns.
Augmentation Increasing a quantity or measure by another quantity. e.g. A plate has 5 cakes, 4 more are added. How many cakes does the plate have now?	**Commutative** Addition and multiplication are commutative, the equation can be carried out in any order.	**Complement** In addition, a number and its complement make a total. e.g 300 is the complement to 700 to make 1000.
Difference The numerical difference between two numbers is found by comparing the quantity in each group.	**Dividend** The number that is divided in a division equation. e.g. 24 ÷ 6 = 4 24 is the dividend.	**Divisor** In division, the number by which another is divided e.g. 24 ÷ 6 = 4 6 is the divisor.
Exchange Change a number or expression for another of equal value.	**Factor** A number that multiplies with another to make a product. e.g. 6 x 4 = 24 6 and 4 are factors of 24	**Minuend** A quantity or number from which another is subtracted. e.g. 20 – 9 = 11 20 is the minuend
Partition To split a number into its component parts.	**Product** The result of multiplying one number by another. e.g. e.g. 6 x 4 = 24 24 is the product	**Quotient** The result of a division. e.g. 24 ÷ 6 = 4 4 is the quotient
Remainder The amount left over after a division when the divisor is not a factor of the dividend. e.g e.g. 25 ÷ 6 = 4 r1 as 6 (the divisor) is not a factor of 25 (the dividend)	**Scaling** Enlarging or reducing a number by a given amount, called the scale factor.	**Subitise** To instantly recognise a number of objects in a small group without needing to count.
Subtrahend A number to be subtracted from another. e.g. 20 – 9 = 11 9 is the subtrahend	**Sum** The result of an addition. e.g. 7+8= 15 15 is the sum	**Total** The sum found by an addition. e.g. 7+8= 15 15 is the total

Further mathematical definitions and explanations can be found at: https://www.mathsisfun.com/definitions/index.html

Progression within Addition

Reception

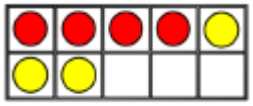
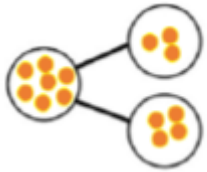
Pupils develop the core ideas that underpin all calculation. They begin by connecting calculation with counting on, developing an understanding of wholes and parts to enable them to calculate efficiently and accurately, and with greater flexibility. Pupils record their calculations in their own ways, **there is no expectation of number sentences at this stage**, however pupils may choose this way to record their thinking. Pupils start to explore addition by sorting groups. They then use sorting to develop their understanding of parts and wholes**.** Pupils combine groups to find the whole, using a part-whole model to support their thinking. They also use the part-whole model to find number bonds within and to 10. Using a five frame and ten frame, pupils add by counting on. They start by finding one more before adding larger numbers using counters or cubes on the frames. Pupils use a number track to add by counting on. Linking this learning to playing board games is an effective way to support pupils' addition.

Key language: count, forwards, backwards, whole, part, recombine, break apart, ones, ten, tens, number bond, add, adding together, addition, plus, total, altogether, part , whole.

Resources and Representations

Five frame, tens frame, Numicon, part whole model, bead strings, cubes.




Why are these representations important?

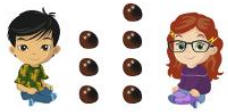
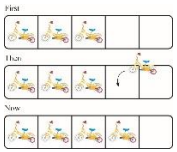
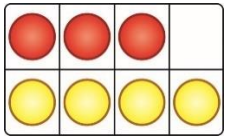
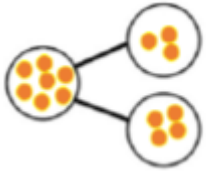
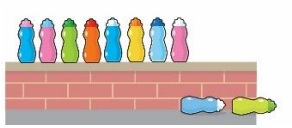
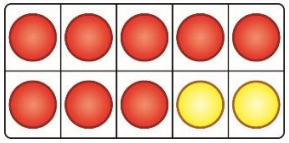
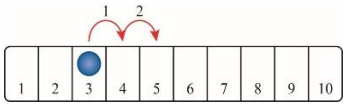
Numicon – when adding numbers together pupils can see how the parts come together to make a whole. Numicon helps pupils to subitise due to their familiarity with the shape of a number.

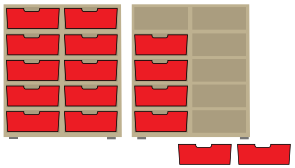
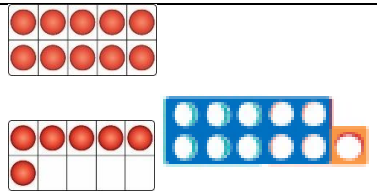
Cubes – cubes support pupils understanding of how parts come together to make a whole. Cubes should only be used when working with smaller numbers; they are inefficient when working with larger numbers as they are difficult to subitise with and pupils may miscount them.

Five and tens frames- these support children to understand the different structures of addition and subtraction. They are particulary useful for deriving number bonds or finding all the ways to make a single digit number.

Part whole model – this helps pupils to develop an understanding of aggregation (parts coming together to form a whole). Part whole models are useful in developing subtraction skills alongside addition. Pupils develop an understanding that when the whole is complete and one of the parts is empty, they must partition the whole to find the remaining part.

Bead strings – these are effective at helping pupils to investigate bonds up to 10 systematically by moving 1 bead at a time to see the different numbers they have partitioned. E.g. 3+2 = 5, move one bead 4+1 = 5. Bead string to 20 work in a similar way but pupils can also grou their beads into fives and use their knowledge of bonds to 10 to help them derive bonds to 20.

Addition	Real-life representation	Other representations
	Counting and adding more (within 5) Pupils add one more person or object to a group to find one more.  *One more than 3 is 4.*	**Counting and adding more (within 5)** Pupils represent first, then, now stories on a five frame. They make the first number and then add one more.  *First, there are 3 bikes.* *Then, 1 more bike came.* *Now, there are 4 bikes.*
	Combining groups to find the whole Pupils sort people and objects into parts and combine them to find the whole.  *The parts are 3 and 4. The whole is 7.*	**Combining groups to find the whole** Pupils use counters or cubes in a part-whole model to find the whole.    *The parts are 3 and 4. The whole is 7.*
	Finding number bonds to 10 Pupils combine two groups to find a number bond to 10.  *There are 8 bottles on the wall.* *There are 2 bottles on the floor.* *There are 10 bottles altogether.*	**Finding number bonds to 10** Use ten frames and part-whole models to represent key number bonds.  *8 and 2 is 10.* *There are 10 altogether.*
	Adding by counting on (number track) Pupils jump along a physical number track. They start at the larger number and count on the smaller number to find the total. 	**Adding by counting on (number track)** Pupils use a number track and a counter. They start at the larger number and count on the smaller number to find the total. 
	Adding by counting on (ten frames) Pupils find the total number by counting on from the larger number.	**Adding by counting on (ten frames)** Pupils make the larger number on the ten frames and then make the smaller number, counting on to find the total. They can use counters, cubes or other objects on the ten frames.

Progression within addition

Key Stage 1

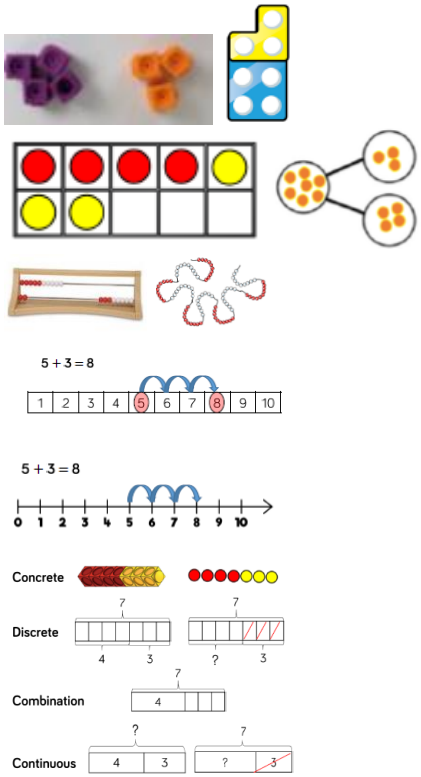
Pupils first learn to connect addition and subtraction with counting, but they soon develop two very important skills: an understanding of parts and wholes, and an understanding of unitising 10s, to develop efficient and effective calculation strategies based on known number bonds and an increasing awareness of place value. Addition and subtraction are taught in a way that is interlinked to highlight the link between the two operations.
A key idea is that pupils will select methods and approaches based on their number sense. Teaching should always emphasise the importance of mathematical thinking to ensure accuracy and flexibility of approach, and the importance of using known number facts to harness their recall of bonds within 20 to support both addition and subtraction methods.
In Year 2, they will start to see calculations presented in a column format, although this is not expected to be formalised until Key Stage 2. It is important that pupils develop mental fluency with addition, rather than relying a formal method for calculations which can / should be carried out mentally.

Key language: whole, part, ones, ten, tens, number bond, add, addition, plus, total, altogether, commutative, exchange

Pupils should be taught the correct vocabulary for the structure of an addition equation: addend + addend = total / sum

Resources and Representations

Five frame, tens frame, Numicon, part whole model, bead strings, rekenrek, cubes, number track, number line, bar model.



Why are these representations important?

Numicon – when adding numbers together pupils can see how the parts come together to make a whole. Numicon helps pupils to subitise due to their familiarity with the shape of a number.

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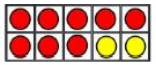
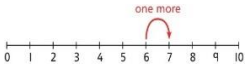
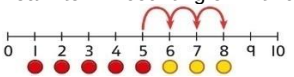
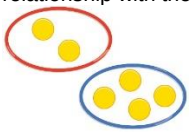
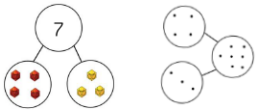
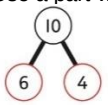
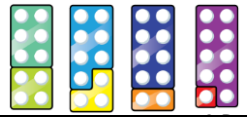
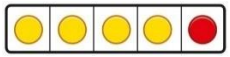
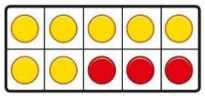
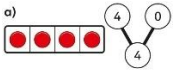
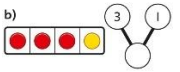
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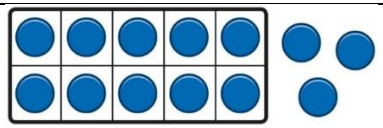
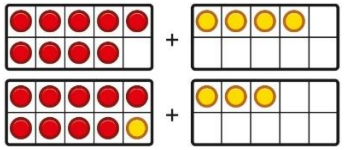
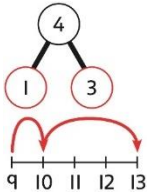
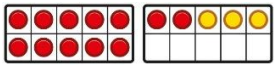
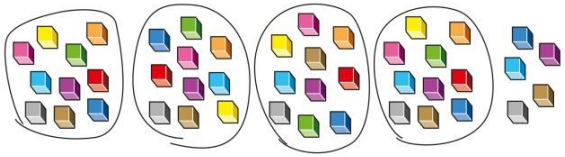
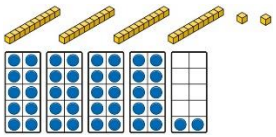
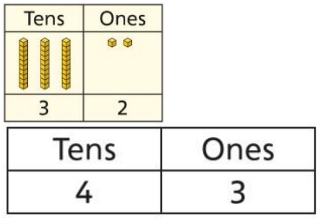
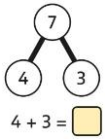
Number track – these are useful to support pupils in their understanding of augmentation (increasing one number by another number) placing a counter on a start number and counting on to find the total.

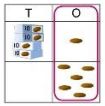
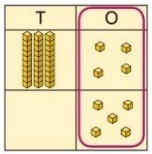
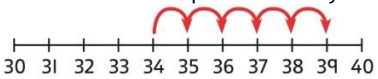
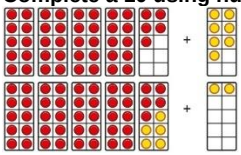
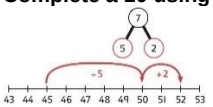
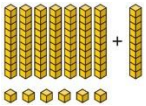
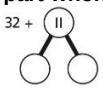
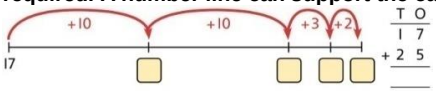
Number lines – these support pupils understanding of augmentation. As pupils progress, they can add to the nearest 10 and then jump to the total.

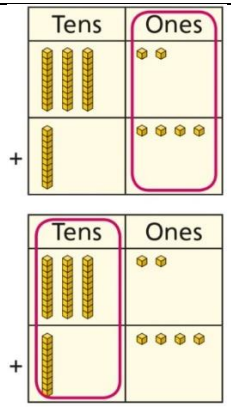
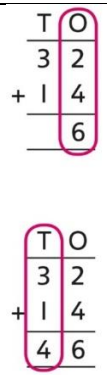
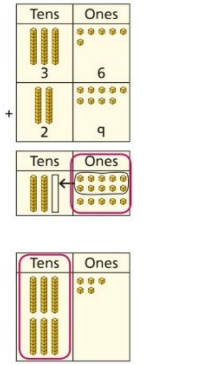
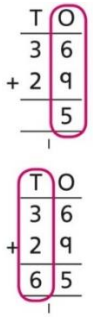
Bar models – these can be made using concrete resources initially before moving onto discrete bar models, where each box represents 1 whole. The combination bar model supports pupils' calculations by counting on from the larger number. It is a good stepping stone towards the continuous bar model. When using a continuous bar model, each rectangle represents a number, the question mark indicates the value to be found.

Year 2 – Place value counters and base 10 may also be used when / if pupils are ready to move towards column addition. Please see LKS2 overview for these representations.

Addition	Concrete	Pictorial	Abstract
Year 1	**Counting and adding more** Pupils add one more person or object to a group to find one more.	**Counting and adding more** Pupils add one more cube or counter to a group to represent one more. *One more than 4 is 5.*   *Pupils use counters or cubes on tens frames.* *2 more than 8 is 10.*	**Counting and adding more** Use a number line to understand how to link counting on with finding one more.  *One more than 6 is 7.* *7 is one more than 6.* Learn to link counting on with adding more than one.  *5 + 3 = 8*
	Understanding part-part-whole relationship Sort people and objects into parts and understand the relationship with the whole.  *The parts are 2 and 4. The whole is 6.*	**Understanding part-part-whole relationship** Pupils draw to represent the parts and understand the relationship with the whole.  *The parts are 1 and 5. The whole is 6.*  The parts are 4 and 3, the whole is 7.	**Understanding part-part-whole relationship** Use a part-whole model to represent the numbers.   *6 + 4 = 10*
	Knowing and finding number bonds within 10 Break apart a group and put back together to find and form number bonds.  *3 + 4 = 7*  *6 = 2 + 4* 	**Knowing and finding number bonds within 10** Use five and ten frames to represent key number bonds.  *5 = 4 + 1*  *10 = 7 + 3*	**Knowing and finding number bonds within 10** Use a part-whole model alongside other representations to find number bonds. Make sure to include examples where one of the parts is zero.   *4 + 0 = 4* *3 + 1 = 4*
	Understanding teen numbers as a complete 10 and some more Complete a group of 10 objects and count more.	**Understanding teen numbers as a complete 10 and some more** Use a ten frame to support understanding of a complete 10 for teen numbers.	**Understanding teen numbers as a complete 10 and some more**. *1 ten and 3 ones equal 13.* *10 + 3 = 13*

	 *13 is 10 and 3 more.*	 *13 is 10 and 3 more.*	
	Bridging the 10 using number bonds Pupils use a bead string to complete a 10 and understand how this relates to the addition.  *7 add 3 makes 10.* *So, 7 add 5 is 10 and 2 more.*	**Bridging the 10 using number bonds** Pupils use counters to complete a ten frame and understand how they can add using knowledge of number bonds to 10.  .	**Bridging the 10 using number bonds** Use a part-whole model and a number line to support the calculation.  *9 + 4 = 13*
	Adding the 1s Pupils use bead strings to recognise how to add the 1s to find the total efficiently.  *2 + 3 = 5* *12 + 3 = 15*	**Adding the 1s** Pupils represent calculations using ten frames to add a teen and 1s.  *2 + 3 = 5* *12 + 3 = 15*	**Adding the 1s** Pupils recognise that a teen is made from a 10 and some 1s and use their knowledge of addition within 10 to work efficiently. *3 + 5 = 8* *So, 13 + 5 = 18*
Addition	**Concrete**	**Pictorial**	**Abstract**
Year 2	**Group objects into 10s and 1s.**  Bundle straws to understand unitising of 10s. 	**Understand 10s and 1s equipment, and link with visual representations on ten frames.** 	**Represent numbers on a place value grid, using equipment or numerals.**  Tens | Ones 3 | 2 Tens | Ones 4 | 3
	Use known bonds and unitising to add 10s.  *I know that 4 + 3 = 7.* *So, I know that 4 tens add 3 tens is 7 tens.*	Use known bonds and unitising to add 10s.  *I know that 4 + 3 = 7.* *So, I know that 4 tens add 3 tens is 7 tens.*	Use known bonds and unitising to add 10s.  *4 + 3 = 7* *4 tens + 3 tens = 7 tens* *40 + 30 = 70*

Adding a 1-digit number to a 2-digit number not bridging a 10	**Add the 1s to find the total. Use known bonds within 10.**  *41 is 4 tens and 1 one.* *41 add 6 ones is 4 tens and 7 ones.* This can also be done in a place value grid.  	**Add the 1s.**  *34 is 3 tens and 4 ones.* *4 ones and 5 ones are 9 ones.* *The total is 3 tens and 9 ones.*  	**Add the 1s.** Understand the link between counting on and using known number facts. Pupils should be encouraged to use known number bonds to improve efficiency and accuracy. 
Adding a 1-digit number to a 2-digit number bridging 10	**Complete a 10 using number bonds.**  *There are 4 tens and 5 ones.* *I need to add 7. I will use 5 to complete a 10, then add 2 more.*	**Complete a 10 using number bonds.** 	**Complete a 10 using number bonds.**  *7 = 5 + 2* *45 + 5 + 2 = 52*
Adding a multiple of 10 to a 2-digit number	**Add the 10s and then recombine.**  *27 is 2 tens and 7 ones.* *50 is 5 tens.* *There are 7 tens in total and 7 ones.* *So, 27 + 50 is 7 tens and 7 ones.*	**Add the 10s and then recombine.**  *66 is 6 tens and 6 ones.* *66 + 10 = 76* A 100 square can support this understanding. 	**Add the 10s and then recombine.** *37 + 20 = ?* *30 + 20 = 50* *50 + 7 = 57* *37 + 20 = 57*
Adding two 2-digit numbers	**Add the 10s and 1s separately.**  *5 + 3 = 8* *There are 8 ones in total.* *3 + 2 = 5* *There are 5 tens in total.* *35 + 23 = 58*	**Add the 10s and 1s separately. Use a part-whole model to support.**  *11 = 10 + 1* *32 + 10 = 42* *42 + 1 = 43* *32 + 11 = 43*	**Add the 10s and the 1s separately, bridging 10s where required. A number line can support the calculations**.  T O 1 7 + 2 5 *17 + 25*
Adding two 2-digit numbers	**Add the 1s. Then add the 10s.**		**Add the 1s. Then add the 10s.**

using a place value grid			
Adding two 2-digit numbers with exchange	**Add the 1s. Exchange 10 ones for a ten. Then add the 10s.** 		**Add the 1s. Exchange 10 ones for a ten. Then add the 10s**. 

Progression within addition

LKS2

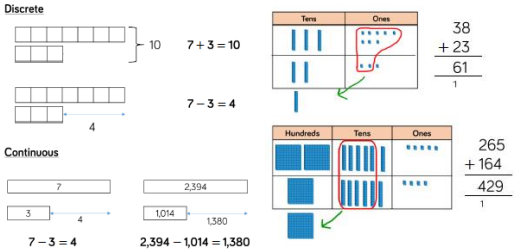
In Years 3 and 4, pupils develop the basis of written methods by building their skills alongside a deep understanding of place value. They should use known addition facts to calculate efficiently and accurately, rather than relying on counting. Pupils use place value equipment to support their understanding, but not as a substitute for thinking. In Year 3, the column methods are built up gradually. Pupils will develop their understanding of how each stage of the calculation, including any exchanges, relates to place value. Pupils should be encouraged to compare mental and written methods for specific calculations, at every stage to make choices about which methods to apply, in order to calculate efficient, without an over reliance on a formal written method. By the end of Year 4, pupils should have developed fluency in column methods alongside a deep understanding, which will allow them to progress confidently in upper Key Stage 2

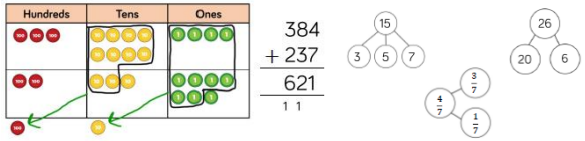
Key language: whole, part, ones, ten, hundreds, thousands, number bond, add, addition, plus, total, altogether, partition, place value, column method, bar model, addend, total, sum of, addend, commutative, exchange.

Pupils should be taught the correct vocabulary for the structure of an addition equation: addend + addend = total /sum

Resources and Representations

Part whole model, place value counters, Base 10 / Dienes, column addition, bar model.



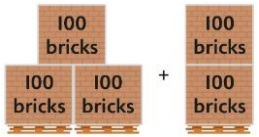
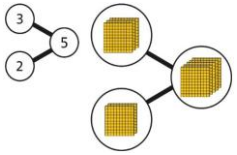
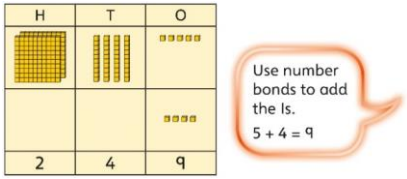
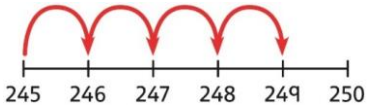
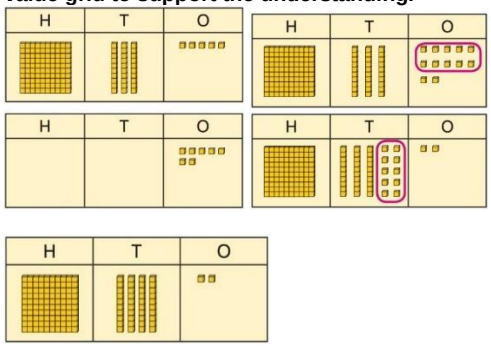
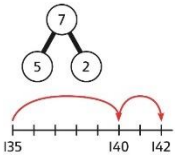


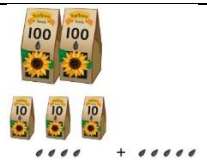
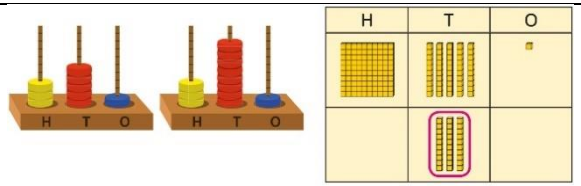
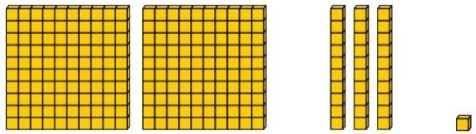
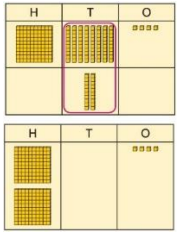
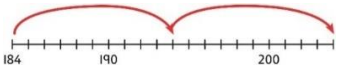
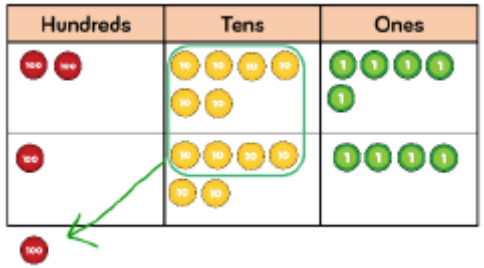
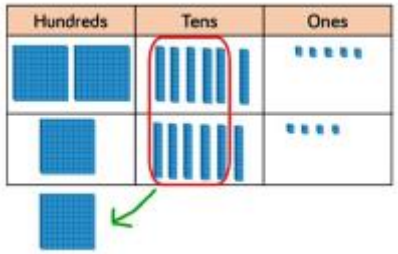
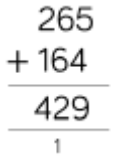
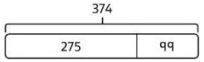
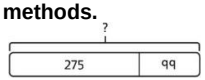
Why are these representations important?

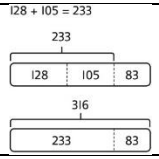
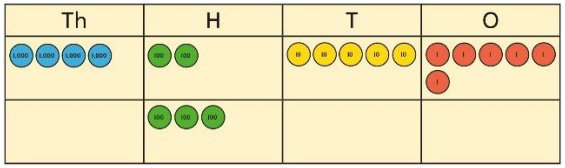
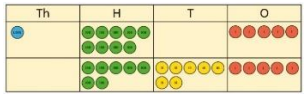
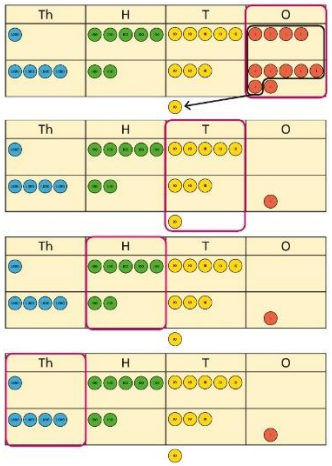
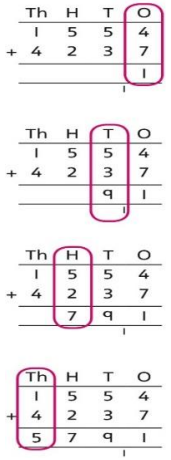
Part whole model – this helps pupils to develop an understanding of aggregation (parts coming together to form a whole). Part whole models are useful in developing subtraction skills alongside addition. Pupils develop an understanding that when the whole is complete and one of the parts is empty, they must partition the whole to find the remaining part.

Bar models – these can be made using concreate resources initially before moving onto discrete bar models, where each box represents 1 whole. The combination bar model supports pupils' calculations by counting on from the larger number. It is a good stepping stone towards the continuous bar model. When using a continuous bar model, each rectangle represents a number, the question mark indicates the value to be found.

Place value counters / Base 10 / Dienes – these are an effective way to support pupils' understanding of column addition. They should first add without exchange before moving on to addition with exchanges. It is important that children write out their calculations alongside their place value / base 10 / dienes model so they can see the clear links with the written method and the model.

Year 3	Concrete	Pictorial	Abstract
Adding 100s	**Use known facts and unitising to add multiples of 100.**  *3 + 2 = 5* *3 hundreds + 2 hundreds = 5 hundreds* *300 + 200 = 500*	**Use known facts and unitising to add multiples of 100.**  *3 + 4 = 7* *3 hundreds + 4 hundreds = 7 hundreds* *300 + 400 = 700*	**Use known facts and unitising to add multiples of 100.** Represent the addition on a number line. Use a part-whole model to support unitising.  *3 + 2 = 5* *300 + 200 = 500*
3-digit number + 1s, no exchange or bridging	**Use number bonds to add the 1s.**  *214 + 4 = ?* *Now there are 4 + 4 ones in total.* *4 + 4 = 8* *214 + 4 = 218*	**Use number bonds to add the 1s.**  *245 + 4* *5 + 4 = 9* *245 + 4 = 249*	**Understand the link with counting on.** *245 + 4*  Use number bonds to add the 1s and understand that this is more efficient and less prone to error. *245 + 4 = ?* *I will add the 1s.* *5 + 4 = 9* *So, 245 + 4 = 249*
3-digit number + 1s with exchange	**Understand that when the 1s sum to 10 or more, this requires an exchange of 10 ones for 1 ten.** Pupils should explore this using unitised objects or physical apparatus.	**Exchange 10 ones for 1 ten where needed. Use a place value grid to support the understanding.**  *135 + 7 = 142*	**Understand how to bridge by partitioning to the 1s to make the next 10.**  *135 + 7 = ?* *135 + 5 + 2 = 142* *Ensure that Pupils understand how to add 1s bridging a 100.* *198 + 5 = ?* *198 + 2 + 3 = 203*
3-digit number + 10s, no exchange	**Calculate mentally by forming the number bond for the 10s.**	**Calculate mentally by forming the number bond for the 10s.** *351 + 30 = ?*	**Calculate mentally by forming the number bond for the 10s.** *753 + 40* *I know that 5 + 4 = 9*

	 *234 + 50* *There are 3 tens and 5 tens altogether.* *3 + 5 = 8* *In total there are 8 tens.* *234 + 50 = 284*	 *5 tens + 3 tens = 8 tens* *351 + 30 = 381*	*So, 50 + 40 = 90* *753 + 40 = 793*
3-digit number + 10s, with exchange	**Understand the exchange of 10 tens for 1 hundred.** 	**Add by exchanging 10 tens for 1 hundred.** *184 + 20 = ?*  *184 + 20 = 204*	**Understand how the addition relates to counting on in 10s across 100.**  *184 + 20 = ?* *I can count in 10s … 194 … 204* *184 + 20 = 204* **Use number bonds within 20 to support efficient mental calculations.** *385 + 50* *There are 8 tens and 5 tens.* *That is 13 tens.* *385 + 50 = 300 + 130 + 5* *385 + 50 = 435*
3-digit number + 2-digit number, exchange required	**Use place value equipment to model addition and understand where exchange is required.** *Use place value counters to represent 265 + 164* 	**Represent the required exchange on a place value grid using equipment.** 	**Use a column method with exchange. Pupils must understand how the method relates to place value at each stage of the calculation.**  **Note: Pupils should also study examples where exchange is required in more than one column, for example *185 + 318 = ?***
Representing addition problems, and selecting appropriate methods	**Encourage Pupils to use their own drawings and choices of place value equipment to represent problems with one or more steps.** These representations will help them to select appropriate methods.	**Pupils understand and create bar models to represent addition problems.** *275 + 99 = ?*  *275 + 99 = 374*	**Use representations to support choices of appropriate methods.**  *I will add 100, then subtract 1 to find the solution.* *128 + 105 + 83 = ?* *I need to add three numbers.*

			
Year 4	**Concrete**	**Pictorial**	**Abstract**
Choosing mental methods where appropriate	**Use unitising and known facts to support mental calculations.** *Make 1,405 from place value equipment.* *Add 2,000.* *Now add the 1,000s.* *1 thousand + 2 thousands = 3 thousands* *1,405 + 2,000 = 3,405*	**Use unitising and known facts to support mental calculations.**  *I can add the 100s mentally.* *200 + 300 = 500* *so, 4,256 + 300 = 4,556*	Use unitising and known facts to support mental calculations. *4,256 + 300 = ?* *2 + 3 = 5* *200 + 300 = 500* *4,256 + 300 = 4,556*
Column addition with exchange	**Use place value equipment on a place value grid to organise thinking.** Ensure that Pupils understand how the columns relate to place value and what to do if the numbers are not all 4-digit numbers. *Use equipment.to show 1,905 + 775.*  *Why have only three columns been used for the second row? Why is the Thousands box empty?* *Which columns will total 10 or more?*	**Use place value equipment to model required exchanges.**  Include examples that exchange in more than one column.	**Use a column method to add, including exchanges.**  Include examples that exchange in more than one column.

Progression within addition

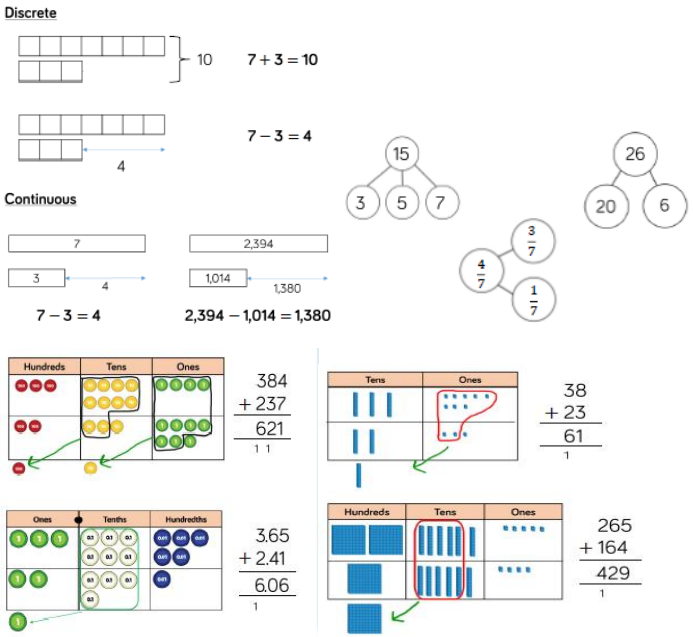
UKS2

Pupils build on their column methods to add and subtract numbers with up to seven digits, and they adapt the methods to calculate efficiently and effectively with decimals, ensuring understanding of place value at every stage.

Pupils compare and contrast methods, and they select mental methods or jottings where appropriate and where these are more likely to be efficient or accurate when compared with formal column methods.
Bar models are used to represent the calculations required to solve problems and may indicate where efficient methods can be chosen.

Key language: decimal, column methods, exchange, partition, mental method, ten thousand, hundred thousand, million, bar model, addend, total, sum of, addend, commutative.
Pupils should be taught the correct vocabulary for the structure of an addition equation: addend + addend = total / sum

Resources and Representations

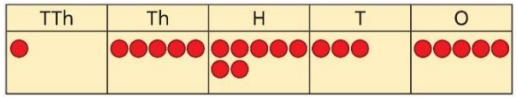
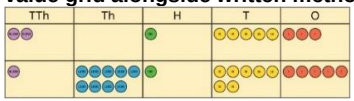
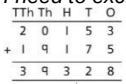
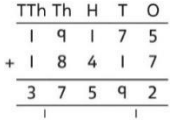
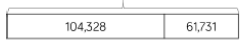
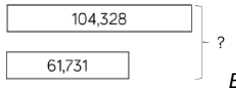
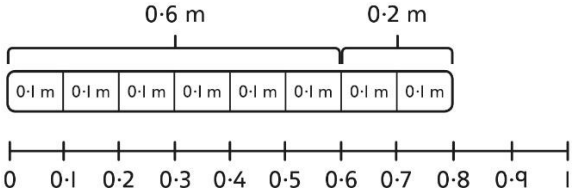
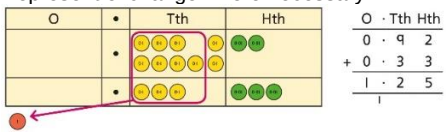
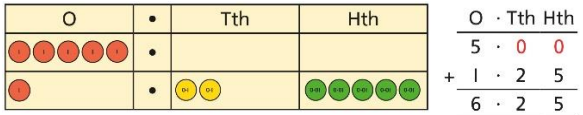
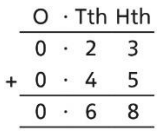
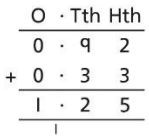


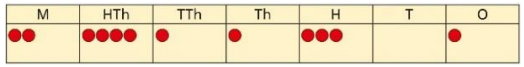
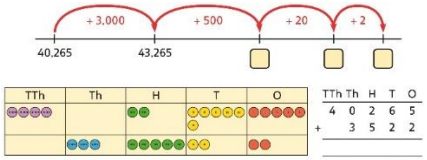
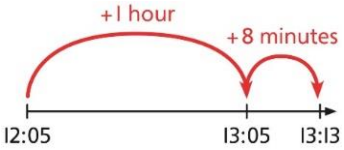
Why are these representations important?

Part whole model – this helps pupils to develop an understanding of aggregation (parts coming together to form a whole). Part whole models are useful in developing subtraction skills alongside addition. Pupils develop an understanding that when the whole is complete and one of the parts is empty, they must partition the whole to find the remaining part.

Bar models – these can be made using concreate resources initially before moving onto discrete bar models, where each box represents 1 whole. The combination bar model supports pupils' calculations by counting on from the larger number. It is a good stepping stone towards the continuous bar model. When using a continuous bar model, each rectangle represents a number, the question mark indicates the value to be found.

Place value counters / Base 10 / Dienes – these are an effective way to support pupils' understanding of column addition. They should first add without exchange before moving on to addition with exchanges. It is important that children write out their calculations alongside their place value / base 10 / dienes model so they can see the clear links with the written method and the model.

Year 5	Concrete	Pictorial	Abstract
Column addition with whole numbers	**Use place value equipment to represent additions.** *Add a row of counters onto the place value grid to show 15,735 + 4,012.* 	**Represent additions, using place value equipment on a place value grid alongside written methods.**  *I need to exchange 10 tens for a 100.* 	**Use column addition, including exchanges.**    *Bar models can be used alongside column addition.*
Adding tenths	**Link measure with addition of decimals.** *Two lengths of fencing are 0·6 m and 0·2 m.* *How long are they when added together?* 	**Use a bar model with a number line to add tenths.**  *0·6 + 0·2 = 0·8* *6 tenths + 2 tenths = 8 tenths*	**Understand the link with adding fractions.** $\frac{6}{10}+\frac{2}{10}=\frac{8}{10}$ *6 tenths + 2 tenths = 8 tenths* *0·6 + 0·2 = 0·8*
Adding decimals using column addition	**Use place value equipment to represent additions.** *Show 0·23 + 0·45 using place value counters.*	**Use place value equipment on a place value grid to represent additions.** Represent exchange where necessary.  Include examples where the numbers of decimal places are different. 	**Add using a column method, ensuring that Pupils understand the link with place value.**  Include exchange where required, alongside an understanding of place value.  Include additions where the numbers of decimal places are different. *3.4 + 0.65 = ?*

			O · Tth Hth 3 · 4 0 + 0 · 6 5 ·
Year 6	**Concrete**	**Pictorial**	**Abstract**
Comparing and selecting efficient methods	**Represent 7-digit numbers on a place value grid, and use this to support thinking and mental methods.** 	**Discuss similarities and differences between methods, and choose efficient methods based on the specific calculation.** Compare written and mental methods alongside place value representations.  Use bar model and number line representations to model addition in problem-solving and measure contexts. 	**Use column addition where mental methods are not efficient. Recognise common errors with column addition.** *32,145 + 4,302 = ?* TTh Th H T O 3 2 1 4 5 + 4 3 0 2 3 6 4 4 7 TTh Th H T O 3 2 1 4 5 + 4 3 0 2 7 5 1 6 5 *Which method has been completed accurately?* *What mistake has been made?* Column methods are also used for decimal additions where mental methods are not efficient. H T O · Tth Hth 1 4 0 · 0 9 + 4 9 · 8 9 1 8 9 · 9 8 1

Progression within subtraction

Reception

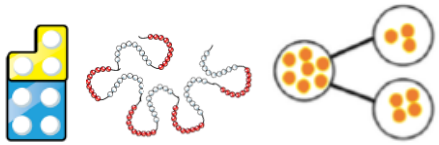
Pupils develop the core ideas that underpin all calculation. They begin by connecting calculation with counting back, developing an understanding of wholes and parts to enable them to calculate efficiently and accurately, and with greater flexibility. Pupils record their calculations in their own ways, **there is no expectation of number sentences at this stage**, however pupils may choose this way to record their thinking.

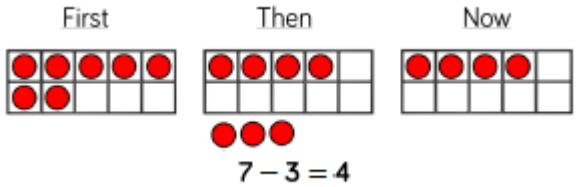
Pupils start to explore subtraction by sorting groups. They use sorting to develop their understanding of parts and wholes. When comparing groups, pupils use the language more than and fewer than. This will lead to finding the difference when they move into KS1. Pupils then connect subtraction with the idea of counting back and finding one less using a five frame to support their thinking. They explore subtraction by breaking apart a whole to find a missing part. This links to their developing recall of number bonds. Pupils count back within 20 using number tracks and ten frames to see the effect of taking away**.**

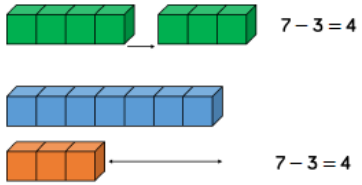
Key language: count, forwards, backwards, whole, part, recombine, break apart, ones, ten, tens, number bond, altogether, first, then, now, subtract, subtraction, find the difference, take away, minus, left, less, more, fewer

Resources and Representations

Five frame, tens frame, Numicon, part whole model, bead strings, cubes.







Why are these representations important?

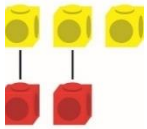
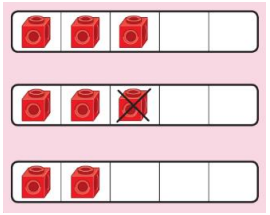
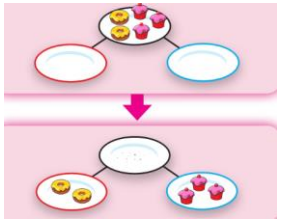
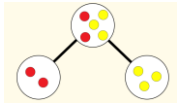
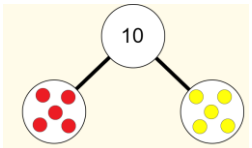
Numicon –pupils can see how the parts come together to make a whole. When subtracting, pupils can start with the whole and then place one of the parts on top of the whole to see what part is missing.

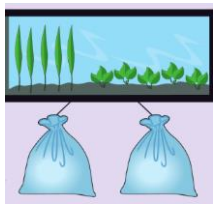
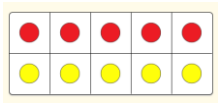
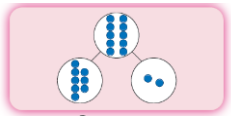
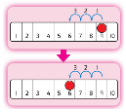
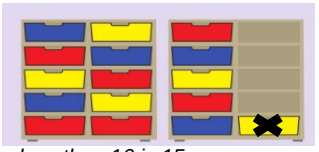
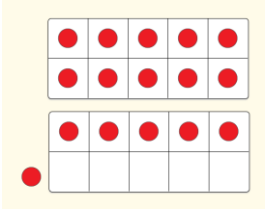
Cubes – pupils can start with the whole and then remove the number of cubes they are subtracting, enabling them to see that subtraction is a reduction. Cubes can also be used to look at subtraction as difference, by lining up the whole and the part you have taken away.

Five and tens frames- these support children to understand the different structures of addition and subtraction. They are useful as a way of introducing 'take away' as they illustrate how the numbers decrease e.g. first there were 7 cars, then 3 cars left, now there are 4 cars.

Part whole model – this helps pupils to develop an understanding of aggregation (parts coming together to form a whole). Part whole models are useful in developing subtraction skills alongside addition. Pupils develop an understanding that when the whole is complete and one of the parts is empty, they must partition the whole to find the remaining part.

Bead strings – by physically moving the beads along the string, pupils are able to develop the concept of subtraction as 'taking away' from the whole. They are able to see that taking away makes the whole smaller.

Reception	Real Life Representations	Other Representations
	Comparing groups Pupils line up objects to compare the amount. They line the objects up either horizontally or vertically.  *Ella has more conkers.* *Tom has fewer conkers.*	**Comparing groups** Pupils line up cubes or counters to compare the amount in each group. Lines can either be horizontal or vertical. A starting line helps to line the objects accurately.  *There are more yellow cubes.* *There are fewer red cubes.*
	Counting back and taking away (within 5) Pupils remove one more person or object from a group to find one less.  *First, there were 3 Pupils.* *Then, 1 child left.* *Now, there are 2 Pupils.*	**Counting back and taking away (within 5)**  Pupils use five frames and objects to make a number. They then remove or cross out one object to find one less. *One less than 3 is 2.*
	Introducing the part-whole model Pupils sort everyday objects into parts.  *One part is the*  *The other part is* 	**Introducing the part-whole model** Pupils use counters or cubes to represent objects in a part-whole model.  *The whole is 5.* *2 is a part. 3 is a part.*
	Finding number bonds to 10 Pupils partition 10 into different groups to find the number bonds to 10.	**Finding number bonds to 10** Pupils use part-whole models, ten frames and counters to find the number bonds to 10.  *10 is the whole.* *5 is a part and 5 is a part.*

	 Pupils begin to work with subtraction number bonds. They break apart 10 to identify different number bonds to 10.  *10 are bouncing.* *2 get off.* *8 are left.* *10 – 2 = 8*	 *10 is the whole.* *5 is a part and 5 is a part.* Pupils use part-whole models, and counters to find missing parts and the subtraction number bonds to 10.  *The parts are 8 and 2.* *10 is the whole.*
	 **Counting back and taking away (number track)** Pupils use game boards and human number tracks to subtract by counting back. *9 take away 3 equals 6* *9…8…7…6*	 **Counting back and taking away (number track)** Pupils use a number track and a counter. They start at the larger number and count back the smaller number to find the answer. *9 take away 3 equals 6* *9…8…7…6*
	Counting back and taking away (ten frames) Pupils count backwards to find one less with numbers up to 20.  *One less than 16 is 15.*	**Counting back and taking away (ten frames)** Pupils remove counters from ten frames to support in counting back with numbers up to 20.  *One less than 16 is 15.*

Progression within subtraction

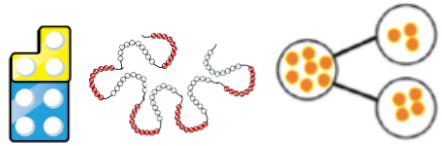
KS1

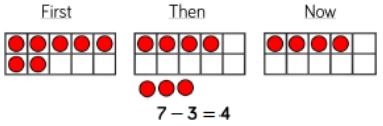
Subtraction Skills Pupils first learn to connect subtraction with counting, moving on to develop two very important skills: an understanding of parts and wholes, and an understanding of unitising 10s, to develop efficient and effective calculation strategies based on known number bonds and an increasing awareness of place value. Addition and subtraction are taught in a way that is interlinked to highlight the link between the two operations.
A key idea is that Pupils will select methods and approaches based on their number sense. For example, in Year 1, when faced with 15 − 3 and 15 − 13, they will adapt their ways of approaching the calculation appropriately. The teaching should always emphasise the importance of mathematical thinking to ensure accuracy and flexibility of approach, and the importance of using known number facts to harness their recall of bonds within 20 to support both addition and subtraction methods.
In Year 2, they will start to see calculations presented in a column format, although this is not expected to be formalised until Key Stage 2. It is important that pupils develop mental fluency with addition, rather than relying a formal method for calculations which can / should be carried out mentally.

Key language: whole, part, ones, ten, subtract, subtraction, find the difference, take away, minus, less, exchange.
Pupils should be taught the correct vocabulary for the structure of a subtraction equation: minuend – subtrahend = difference

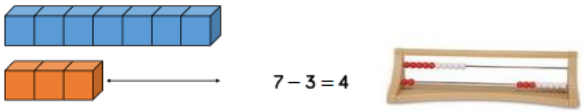
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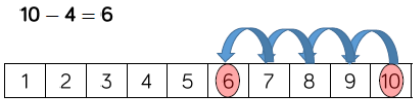
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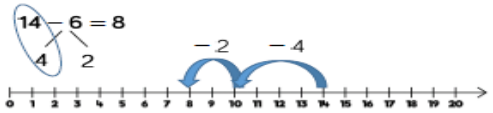












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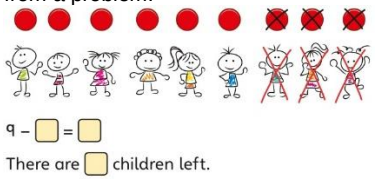
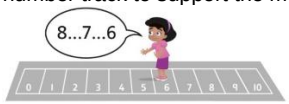
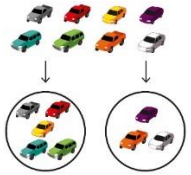
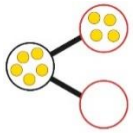
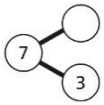
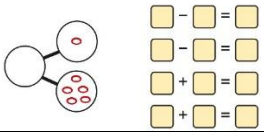
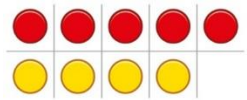
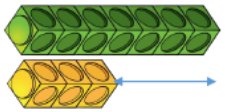
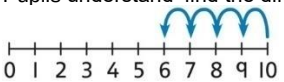
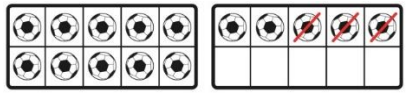
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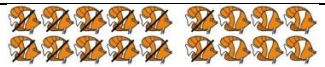
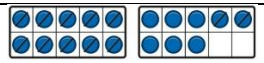
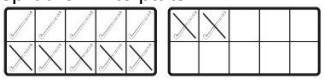
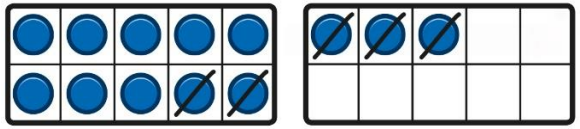
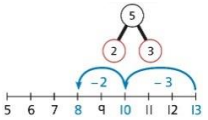
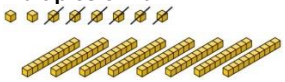
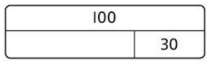
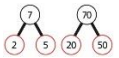
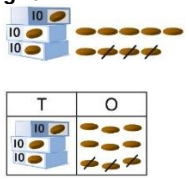
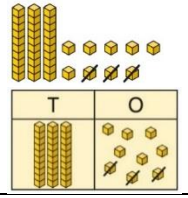
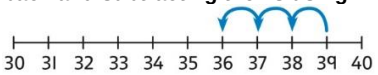
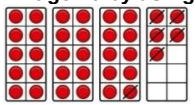
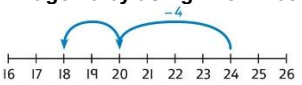
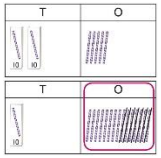
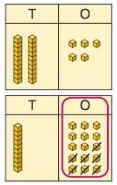
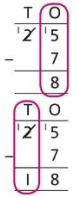
Part whole model – this helps pupils to develop an understanding of aggregation (parts coming together to form a whole). Part whole models are useful in developing subtraction skills alongside addition. Pupils develop an understanding that when the whole is complete and one of the parts is empty, they must partition the whole to find the remaining part.

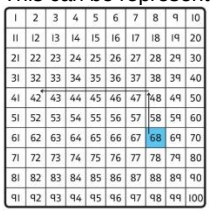
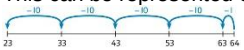
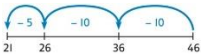
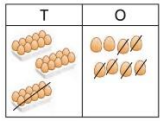
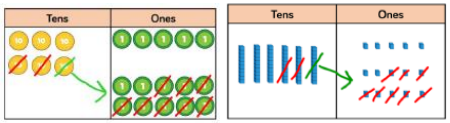
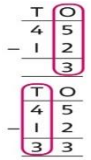
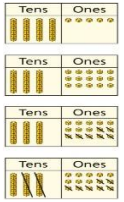
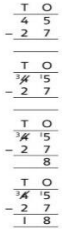
Bead strings and rekenreks – by physically moving the beads along the string, pupils are able to develop the concept of subtraction as 'taking away' from the whole. They are able to see that taking away makes the whole smaller. When using a rekenrek pupils can compare the top and bottom rows to investigate difference.

Number tracks and number lines – when subtracting pupils count back to find their answer. Starting at the minuend and taking away the subtrahend to find the difference between numbers. Pupils should be encouraged to partition the subtrahend when subtracting, by jumping to the previous ten first.

Year 2 – Place value counters and base 10 may also be used when / if pupils are ready to move towards written subtraction methods. Please see LKS2 overview for these representations.

Year 1	Concrete	Pictorial	Abstract
	Counting back and taking away Pupils arrange objects and remove to find how many are left.  *1 less than 6 is 5.* *6 subtract 1 is 5.*	**Counting back and taking away** Pupils draw and cross out or use counters to represent objects from a problem.  9 – ☐ = ☐ There are ☐ children left.	**Counting back and taking away** Pupils count back to take away and use a number line or number track to support the method.  *9 − 3 = 6*
	Finding a missing part, given a whole and a part Pupils separate a whole into parts and understand how one part can be found by subtraction.  *8 − 5 = ?*	**Finding a missing part, given a whole and a part** Pupils represent a whole and a part and understand how to find the missing part by subtraction.  5 – 4 = ☐	**Finding a missing part, given a whole and a part** Pupils use a part-whole model to support the subtraction to find a missing part.  *7 − 3 = ?* Pupils develop an understanding of the relationship between addition and subtraction facts in a part-whole model. 
	Finding the difference Arrange two groups so that the difference between the groups can be worked out.  *8 is 2 more than 6.* *6 is 2 less than 8.* *The difference between 8 and 6 is 2.*	**Finding the difference** Represent objects using sketches or counters to support finding the difference.  *5 − 4 = 1* *The difference between 5 and 4 is 1.* 	**Finding the difference** Pupils understand 'find the difference' as subtraction.  *10 − 4 = 6* *The difference between 10 and 6 is 4.*
	Subtraction within 20 Understand when and how to subtract 1s efficiently. Use a bead string to subtract 1s efficiently.  *5 − 3 = 2* *15 − 3 = 12*	**Subtraction within 20** Understand when and how to subtract 1s efficiently.  *5 − 3 = 2* *15 − 3 = 12*	**Subtraction within 20** Understand how to use knowledge of bonds within 10 to subtract efficiently. *5 − 3 = 2* *15 − 3 = 12*
	Subtracting 10s and 1s For example: 18 − 12 Subtract 12 by first subtracting the 10, then the remaining 2.	**Subtracting 10s and 1s** For example: 18 − 12 Use ten frames to represent the efficient method of subtracting 12.	**Subtracting 10s and 1s** Use a part-whole model to support the calculation.

	 *First subtract the 10, then take away 2.*	 *First subtract the 10, then subtract 2.*	 *19 − 14* *19 − 10 = 9* *9 − 4 = 5* *So, 19 − 14 = 5*
	Subtraction bridging 10 using number bonds For example: 12 − 7 Arrange objects into a 10 and some 1s, then decide on how to split the 7 into parts.  *7 is 2 and 5, so I take away the 2 and then the 5.*	**Subtraction bridging 10 using number bonds** Represent the use of bonds using ten frames.  *For 13 – 5, I take away 3 to make 10, then take away 2 to make 8.*	**Subtraction bridging 10 using number bonds** Use a number line and a part-whole model to support the method. *13 − 5* 
Year 2	**Concrete**	**Pictorial**	**Abstract**
Subtracting multiples of 10	**Use known number bonds and unitising to subtract multiples of 10.**  *8 subtract 6 is 2.* *So, 8 tens subtract 6 tens is 2 tens.*	**Use known number bonds and unitising to subtract multiples of 10.**  *10 − 3 = 7* *So, 10 tens subtract 3 tens is 7 tens.*	**Use known number bonds and unitising to subtract multiples of 10.**  *7 tens subtract 5 tens is 2 tens.* *70 − 50 = 20*
Subtracting a single-digit number	**Subtract the 1s. This may be done in or out of a place value grid.** 	**Subtract the 1s. This may be done in or out of a place value grid.** 	**Subtract the 1s. Understand the link between counting back and subtracting the 1s using known bonds.**  *9 − 3 = 6* *39 − 3 = 36*
Subtracting a single-digit number bridging 10	**Bridge 10 by using known bonds.**  *35 − 6* *I took away 5 counters, then 1 more.*	**Bridge 10 by using known bonds.**  *35 − 6* *First, I will subtract 5, then 1.*	**Bridge 10 by using known bonds.**  *24 − 6 = ?* *24 − 4 − 2 = ?*
Subtracting a single-digit number using exchange	**Exchange 1 ten for 10 ones. This may be done in or out of a place value grid.** 	**Exchange 1 ten for 10 ones.** 	**Exchange 1 ten for 10 ones.**  *25 − 7 = 18* *This method should not replace efficient mental strategies.*

Subtracting a 2-digit number	**Subtract by taking away.**  *61 − 18* *I took away 1 ten and 8 ones.*	**Subtract the 10s and the 1s.** This can be represented on a 100 square. 	**Subtract the 10s and the 1s**. This can be represented on a number line.  *64 − 41 = ?* *64 − 1 = 63* *63 − 40 = 23* *64 − 41 = 23*  *46 − 20 = 26* *26 − 5 = 21* *46 − 25 = 21*
Subtracting a 2-digit number using place value and columns	**Subtract the 1s. Then subtract the 10s. This may be done in or out of a place value grid.**  *38 − 16 = 22*	**Subtract the 1s. Then subtract the 10s.** 	**Using column subtraction, subtract the 1s. Then subtract the 10s.** 
Subtracting a 2-digit number with exchange		**Exchange 1 ten for 10 ones. Then subtract the 1s. Then subtract the 10s.** 	**Using column subtraction, exchange 1 ten for 10 ones. Then subtract the 1s. Then subtract the 10s.** 

Progression within subtraction

LKS2

Subtraction Skills: Year 3, the column method for subtraction is built up gradually. Pupils will develop their understanding of how each stage of the calculation, including any exchanges, relates to place value. Pupils should be encouraged to compare mental and written methods for specific calculations, at every stage to make choices about which methods to apply, in order to calculate efficient, without an over reliance on a formal written method.

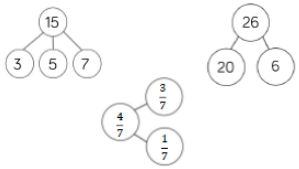
By the end of Year 4, pupils should have developed fluency in column methods alongside a deep understanding, which will allow them to progress confidently in upper Key Stage 2.

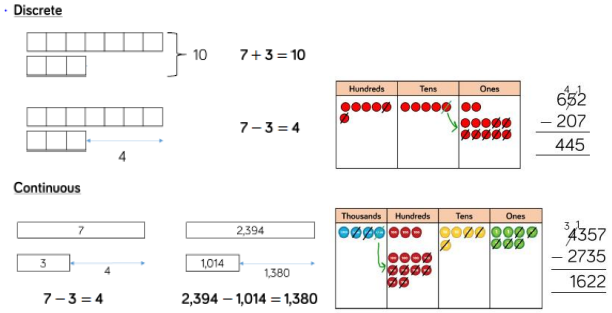
Key language: whole, part, ones, ten, hundreds, thousands, partition, place value, column method, bar model, minuend, subtrahend, difference, inverse, exchange.

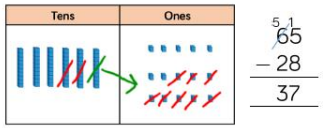
Pupils should be taught the correct vocabulary for the structure of a subtraction equation: minuend – subtrahend = difference

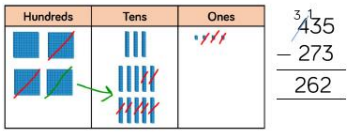
Resources and Representations

Part-whole model, bar model, base 10, place value counters, dienes, column addition







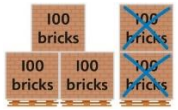
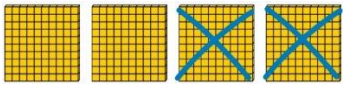
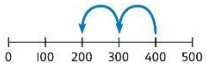
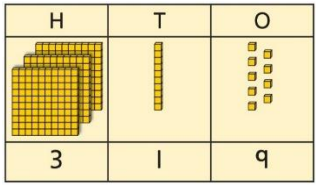
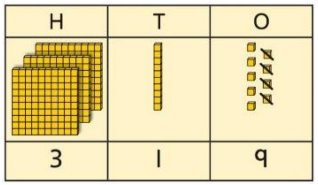
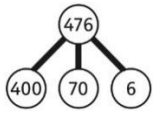
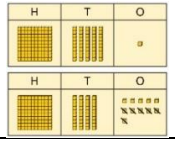
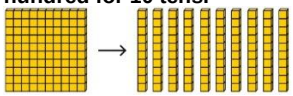
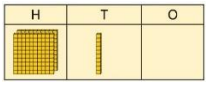
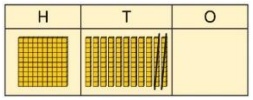
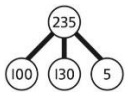


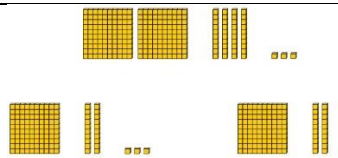
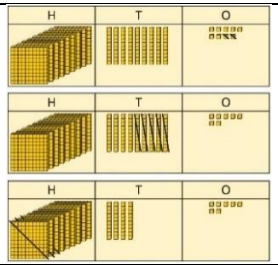
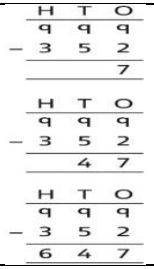
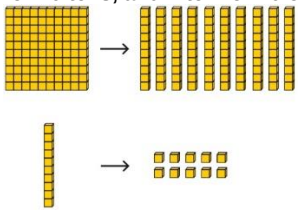
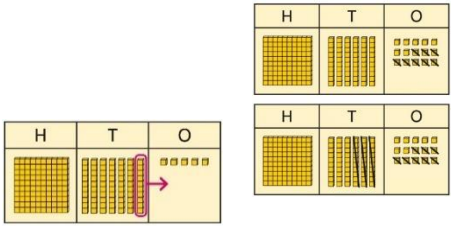
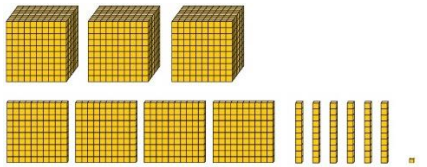
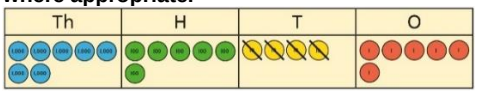
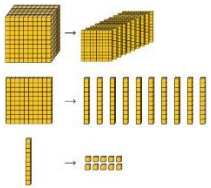
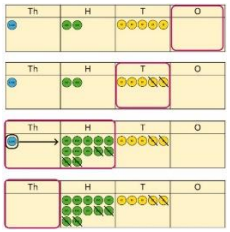
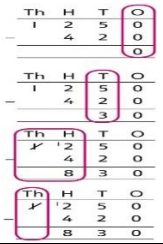
Why are these representations important?

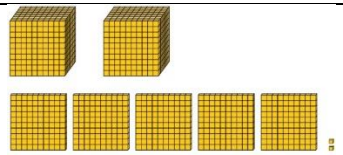
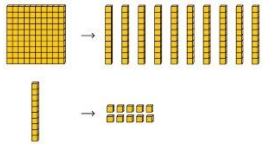
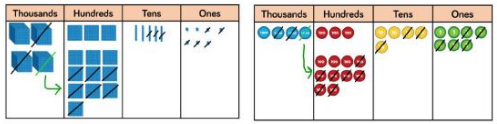
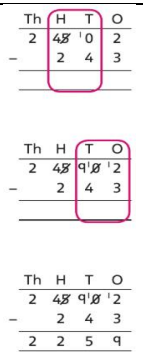
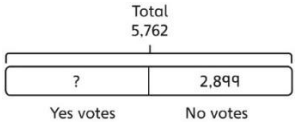
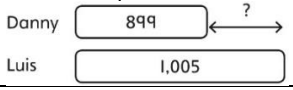
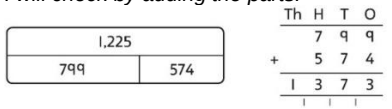
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It is important that children write out their calculations alongside their place value / base 10 / dienes model so they can see the clear links with the written method and the model.
Place value counters are more efficient then base 10 when calculating with larger numbers or decimals.

Year 3	Concrete	Pictorial	Abstract
Subtracting 100s	**Use known facts and unitising to subtract multiples of 100.**  *5 − 2 = 3* *500 − 200 = 300*	**Use known facts and unitising to subtract multiples of 100.**  *4 − 2 = 2* *400 − 200 = 200*	**Understand the link with counting back in 100s.**  *400 − 200 = 200* Use known facts and unitising as efficient and accurate methods. *I know that 7 − 4 = 3. Therefore, I know that 700 − 400 = 300.*
3-digit number − 1s, no exchange	**Use number bonds to subtract the 1s.**  *214 − 3 = ?*  *4 − 3 = 1* *214 − 3 = 211*	**Use number bonds to subtract the 1s.**  *319 − 4 = ?*  *9 − 4 = 5* *319 − 4 = 315*	**Understand the link with counting back using a number line.** Use known number bonds to calculate mentally. *476 − 4 = ?*  *6 − 4 = 2* *476 − 4 = 472*
3-digit number − 1s, exchange or bridging required	**Understand why an exchange is necessary by exploring why 1 ten must be exchanged.** Use place value equipment.	**Represent the required exchange on a place value grid.** *151 − 6 = ?*  	**Calculate mentally by using known bonds.** *151 − 6 = ?* *151 − 1 − 5 = 145*
3-digit number − 10s, exchange or bridging required	**Use equipment to understand the exchange of 1 hundred for 10 tens.** 	**Represent the exchange on a place value grid using equipment.** *210 − 20 = ?*  *I need to exchange 1 hundred for 10 tens, to help subtract 2 tens.*  *210 − 20 = 190*	**Understand the link with counting back on a number line.** Use flexible partitioning to support the calculation. *235 − 60 = ?*  *235 = 100 + 130 + 5* *235 − 60 = 100 + 70 + 5* *= 175*
3-digit number − up to 3-digit number	**Use place value equipment to explore the effect of splitting a whole into two parts, and understand the link with taking away.**	**Represent the calculation on a place value grid.**	**Use column subtraction to calculate accurately and efficiently.**

			
3-digit number − up to 3-digit number, exchange required	**Use equipment to enact the exchange of 1 hundred for 10 tens, and 1 ten for 10 ones.** 	**Model the required exchange on a place value grid**. *175 − 38 = ?* *I need to subtract 8 ones, so I will exchange a ten for 10 ones.* 	**Use column subtraction to work accurately and efficiently.**  175 − 38 = 137 If the subtraction is a 3-digit number subtract a 2-digit number, pupils should understand how the recording relates to the place value, and so how to line up the digits correctly. Pupils should also understand how to exchange in calculations where there is a zero in the 10s column. 
Year 4	**Concrete**	**Pictorial**	**Abstract**
Choosing mental methods where appropriate	**Use place value equipment to justify mental methods.**  *What number will be left if we take away 300?*	**Use place value grids to support mental methods where appropriate.**  *7,646 − 40 = 7,606*	**Use knowledge of place value and unitising to subtract mentally where appropriate.** *3,501 − 2,000* *3 thousands − 2 thousands = 1 thousand* *3,501 − 2,000 = 1,501*
Column subtraction with exchange	**Understand why exchange of a 1,000 for 100s, a 100 for 10s, or a 10 for 1s may be necessary.** 	**Represent place value equipment on a place value grid to subtract, including exchanges where needed.** 	**Use column subtraction, with understanding of the place value of any exchange required.** 
Column subtraction with exchange across more than one column	**Understand why two exchanges may be necessary.** *2,502 − 243 = ?*	**Make exchanges across more than one column where there is a zero as a place holder.**	**Make exchanges across more than one column where there is a zero as a place holder.** *2,502 − 243 = ?*

	 *I need to exchange a 10 for some 1s, but there are not any 10s here.* 	4,357 − 2,735 = 1,622 	
Representing subtractions and checking strategies **These should be modelled alongside the column method.**		**Use bar models to represent subtractions where a part needs to be calculated.**  *I can work out the total number of Yes votes using 5,762 − 2,899.* Bar models can also represent 'find the difference' as a subtraction problem. 	**Use inverse operations to check subtractions.** *I calculated 1,225 − 799 = 574.* *I will check by adding the parts.*  *The parts do not add to make 1,225.* *I must have made a mistake.*

Progression within subtraction

UKS2

Subtraction Skills: Pupils build on their column methods to subtract numbers with up to seven digits, and they adapt the methods to calculate efficiently and effectively with decimals, ensuring understanding of place value at every stage.

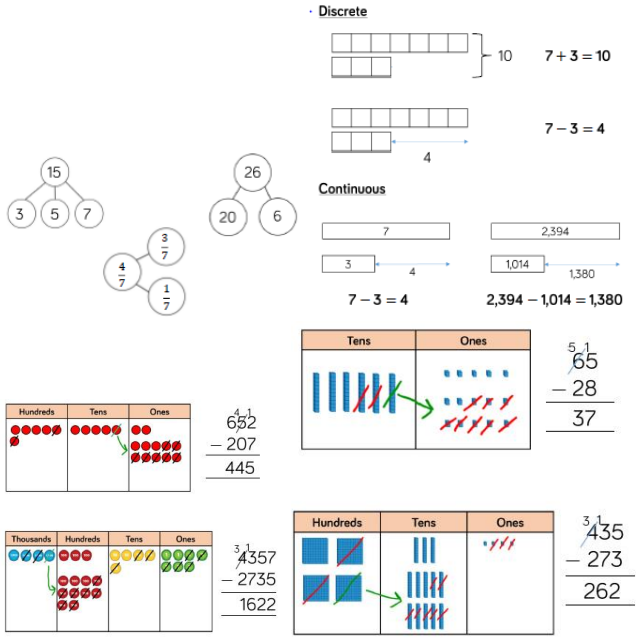
Pupils compare and contrast methods, and they select mental methods or jottings where appropriate and where these are more likely to be efficient or accurate when compared with formal column methods.

Bar models are used to represent the calculations required to solve problems and may indicate where efficient methods can be chosen.

Key language: whole, part, ones, ten, hundreds, thousands, number bond, altogether, partition, place value, column method, bar model, minuend, subtrahend, difference, minus, subtract, less than, fewer than, decimal, column methods, exchange, mental method, ten thousand, hundred thousand, million.

Pupils should be taught the correct vocabulary for the structure of a subtraction equation: minuend – subtrahend = difference

Resources and Representations

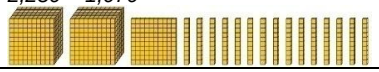


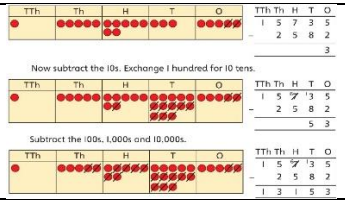
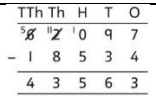
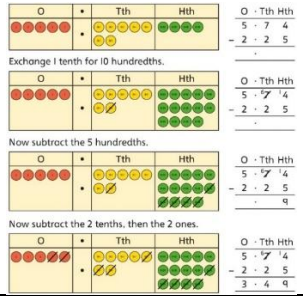
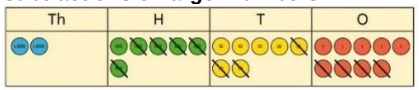
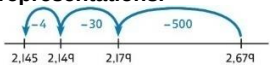
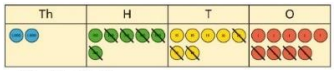
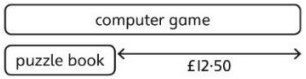
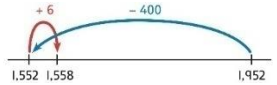
Why are these representations important?

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Place value counters are more efficient then base 10 when calculating with larger numbers or decimals.

Year 5	Concrete	Pictorial	Abstract
Column subtraction with whole numbers	**Use place value equipment to understand where exchanges are required.** *2,250 – 1,070* 	**Represent the stages of the calculation using place value equipment on a grid alongside the calculation, including exchanges where required.** *15,735 − 2,582 = 13,153*	**Use column subtraction methods with exchange where required.**

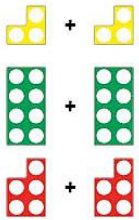
			 *62,097 − 18,534 = 43,563*
Subtracting decimals	**Explore complements to a whole number by working in the context of length.**  1 m – ☐ m = ☐ m *1 − 0·49 = ?*	**Use a place value grid to represent the stages of column subtraction, including exchanges where required.** *5·74 − 2·25 = ?* 	**Use column subtraction, with an understanding of place value, including subtracting numbers with different numbers of decimal places.** *3·921 − 3·75 = ?* O · Tth Hth Thth 3 · 9 2 1 − 3 · 7 5 0 .
Year 6	**Concrete**	**Pictorial**	**Abstract**
Comparing and selecting efficient methods	**Use counters on a place value grid to represent subtractions of larger numbers.** 	**Compare subtraction methods alongside place value representations.**   Th H T O 2 6 7 9 − 5 3 4 2 1 4 5 Use a bar model to represent calculations, including ‘find the difference’ with two bars as comparison. 	**Compare and select methods.** **Use column subtraction when mental methods are not efficient.** **Use two different methods for one calculation as a checking strategy.** Th H T O 1 8 14 12 − 1 5 5 8 3 9 4  Use column subtraction for decimal problems, including in the context of measure. H T O · Tth Hth 3 0 9 · 6 0 − 2 0 6 · 4 0 1 0 3 · 2 0

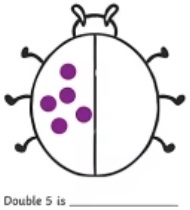
Progression within Multiplication

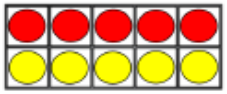
Reception

Pupils develop the core ideas that underpin all calculation. They begin by connecting calculation with counting on, developing an understanding of wholes and parts to enable them to calculate efficiently and accurately, and with greater flexibility. Pupils record their calculations in their own ways, **there is no expectation of number sentences at this stage**, however Pupils may choose this way to record their thinking.
Pupils first start to look at the idea of equal groups through their exploration of doubles. They use five frames and objects to check that groups are equal.
They highlight patterns between doubling and halving seeing that double 2 is 4 and half of 4 is 2.

Key Vocabulary: ones, group, share, equal.

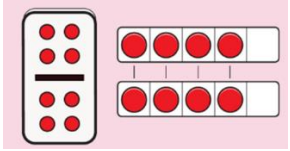






Why are these representations important?

Counters and concrete objects – pupils need lots of experience of putting objects into equal groups – the foundation for multiplication.

Reception	Real-life representation	Other representations
Multiplication	**Making doubles** Pupils explore doubles in their environment including in games such as on dominoes or dice. They focus on the understanding of doubles being 2 equal groups.  *Double 4 is 8.* *Double 2 is 4.* *Double 3 is 6.*	**Making doubles** Pupils use five frames to find doubles by lining up counters or cubes.  *Double 4 is 8.*

Progression within Multiplication

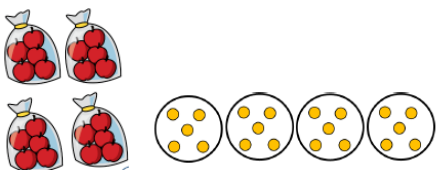
KS1

Pupils develop an awareness of equal groups and link this with counting in equal steps, starting with 2s, 5s and 10s. In Year 2, they learn to connect the language of equal groups with the mathematical symbol for multiplication.
They learn how multiplication can be related to repeated addition.
Pupils begin to recall some key multiplication facts, including doubles, and an understanding of the 2, 5 and 10 times-tables and how they are related to counting.
In this key stage, it is essential that pupils explore and experience a variety of strong images and manipulative representations of equal groups, including concrete experiences as well as abstract calculations.

Key language: whole, part, ones, altogether, equal groups, times, multiply, times-table, factor, product

Pupils should be taught the correct vocabulary for the structure of a multiplication equation: factor X factor = product

Resources and Representations





5 + 5 + 5 + 5 = 20
4 × 5 = 20
5 × 4 = 20



5 × 4 = 20
4 × 5 = 20



5 × 4 = 20
4 × 5 = 20



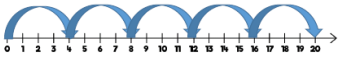
5 × 3 = 15
3 × 5 = 15

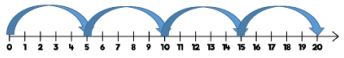
15 ÷ 3 = 5





6 × 3 = 18
3 × 6 = 18





4 × 5 = 20
5 × 4 = 20

Why are these representations important?

Counters and concrete objects – pupils need lots of experience of putting objects into equal groups, building their understanding that by counting each group as a whole, is the beginning of multiplication.

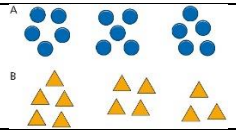
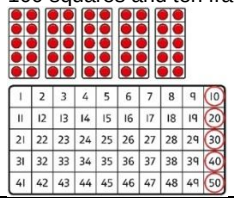
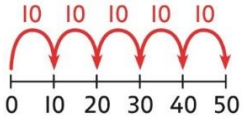
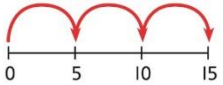
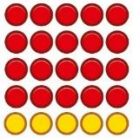
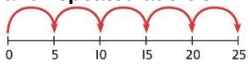
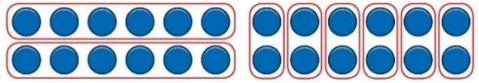
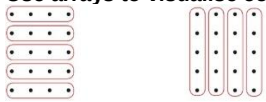
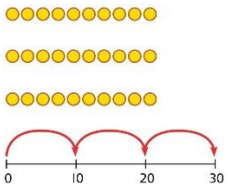
Arrays - these can be used for building multiplication facts in a meaningful way. Before memorising times tables, children use arrays to develop their understanding of how these facts are derived. E.g. by progressively adding another column of 4 objects, children can build the 4 x table for themselves. Arrays provides a visual image for children to draw upon as they begin to use and memorise the basic number facts.

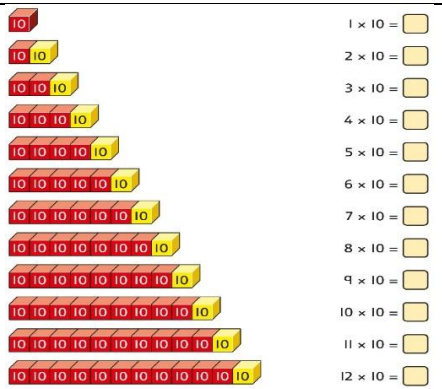
Numicon – this helps to support pupils' understanding of multiplication as repeated addition. Pupils can build multiplications in rows, they can then use the ten Numicon shape over the top of the row to check totals. Numicon helps pupils explore patterns in multiplication e.g. odd x odd = even, odd x even = odd, even x even = even.

Bead strings – working with bead strings to 100 helps pupils to develop their understanding of multiplication as repeated addition. Pupils should be encouraged to count in multiples as they make their groups e.g. 4 x 6 "4, 8,12,16,20,24". The colour of beads supports pupils in seeing how many groups of 10 they have to calculate more efficiently.

Number Tracks and Number Lines – these are useful to support pupils to count forwards in multiples. Moving counters (translucent ones are best) along the number track can support pupils to keep a track of their counting. When multiplying, children place their counter on 0 to start and then count on to find the product of numbers. Number lines and number tracks are useful with smaller multiples, however they are less efficient as numbers become larger due to the required size of the number line.

Year 1	Concrete	Pictorial	Abstract
	Recognising and making equal groups Children arrange objects in equal and unequal groups and understand how to recognise whether they are equal.	**Recognising and making equal groups** Children draw and represent equal and unequal groups.	**Describe equal groups using words** *Three equal groups of 4.* *Four equal groups of 3.*

			
	Finding the total of equal groups by counting in 2s, 5s and 10s  There are 5 pens in each pack … 5…10…15…20…25…30…35…40…	**Finding the total of equal groups by counting in 2s, 5s and 10s** 100 squares and ten frames support counting in 2s, 5s and 10s. 	**Finding the total of equal groups by counting in 2s, 5s and 10s** Use a number line to support repeated addition through counting in 2s, 5s and 10s. 
Year 2	**Concrete**	**Pictorial**	**Abstract**
Equal groups and repeated addition	**Recognise equal groups and write as repeated addition and as multiplication.**  *3 groups of 5 chairs* *15 chairs altogether*	**Recognise equal groups using standard objects such as counters and write as repeated addition and multiplication.**  *3 groups of 5* *15 in total*	**Use a number line and write as repeated addition and as multiplication.**  *5 + 5 + 5 = 15* *3 × 5 = 15*
Using arrays to represent multiplication and support understanding	**Understand the relationship between arrays, multiplication and repeated addition.**  *4 groups of 5*	**Understand the relationship between arrays, multiplication and repeated addition.**  *4 groups of 5 … 5 groups of 5*	**Understand the relationship between arrays, multiplication and repeated addition.**  *5 × 5 = 25*
Understanding commutativity	**Use arrays to visualise commutativity.**  *I can see 6 groups of 3.* *I can see 3 groups of 6.*	**Form arrays using counters to visualise commutativity. Rotate the array to show that orientation does not change the multiplication.**  *This is 2 groups of 6 and also 6 groups of 2.*	**Use arrays to visualise commutativity.**  *4 + 4 + 4 + 4 + 4 = 20* *5 + 5 + 5 + 5 = 20* *4 × 5 = 20 and 5 × 4 = 20*
Learning ×2, ×5 and ×10 table facts	**Develop an understanding of how to unitise groups of 2, 5 and 10 and learn corresponding times-table facts.**	**Understand how to relate counting in unitised groups and repeated addition with knowing key times-table facts.**  ***10 + 10 + 10 = 30*** ***3 × 10 = 30***	**Understand how the times-tables increase and contain patterns.**

	   ***3 groups of 10 … 10, 20, 30*** ***3 × 10 = 30***		 ***5 × 10 = 50*** ***6 × 10 = 60***

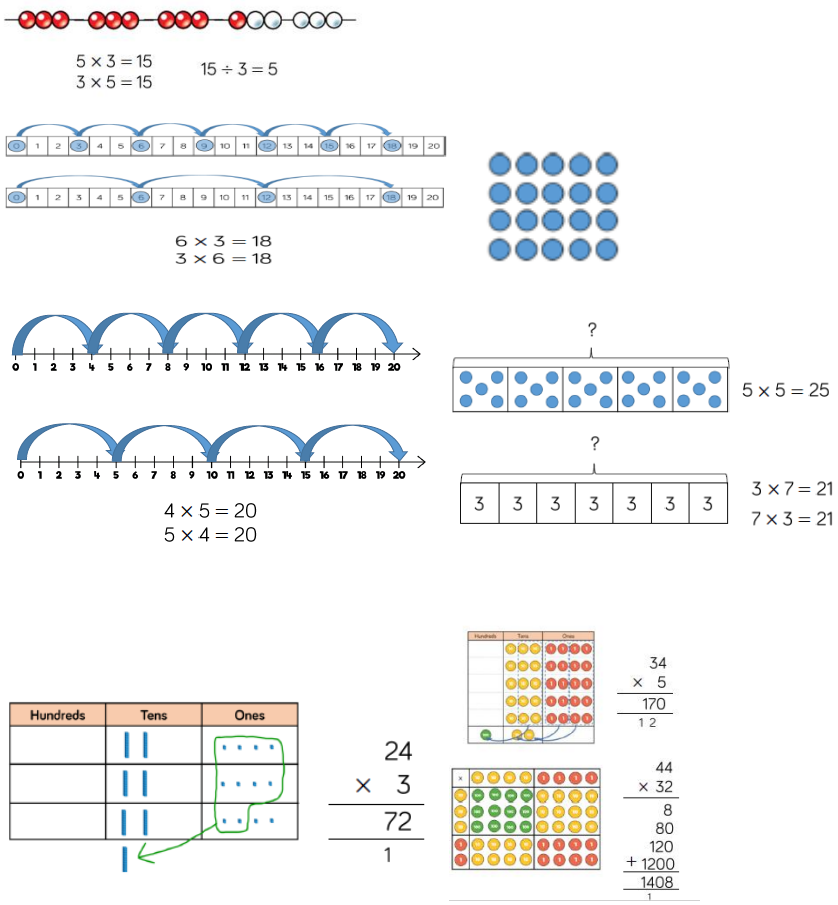
LKS2

Progression within Multiplication

Pupils build a solid grounding in times-tables, understanding the multiplication and division facts in tandem. As such, they should be as confident knowing that 35 divided by 7 is 5 as knowing that 5 times 7 is 35.
Pupils develop key skills to support multiplication methods: unitising, commutativity, and how to use partitioning effectively.
Unitising allows pupils to use known facts to multiply multiples of 10 and 100 efficiently.
Commutativity gives pupils flexibility in applying known facts to calculations and problem solving.
An understanding of partitioning allows pupils to extend their skills to multiplying 2- and 3-digit numbers by a single digit.
Pupils develop column methods to support multiplications in these cases.

Key language: partition, place value, tens, hundreds, thousands, column method, whole, part, equal groups, multiple, multiple, multiplier, multiplicand, product
Pupils should be taught the correct vocabulary for the structure of a multiplication equation: factor x factor = product

Resources and Representations



Why are these representations important?

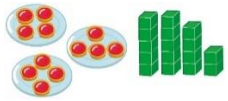
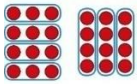
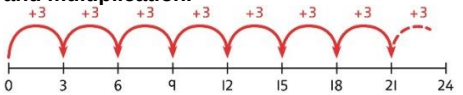
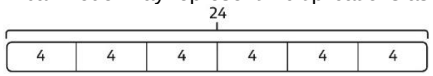
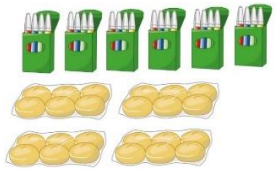
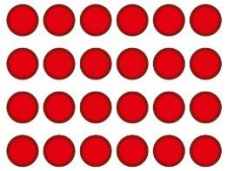
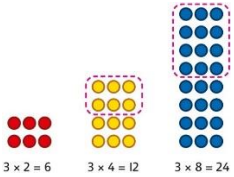
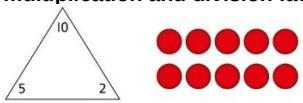
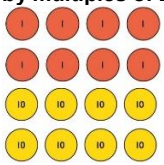
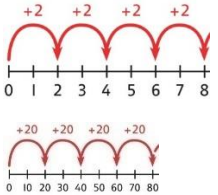
Arrays - these can be used for building multiplication facts in a meaningful way. Before memorising times tables, children use arrays to develop their understanding of how these facts are derived. E.g. by progressively adding another column of 4 objects, children can build the 4 x table for themselves. Arrays provides a visual image for children to draw upon as they begin to use and memorise the basic number facts. They also help develop an understanding of commutativity.
Bead strings – working with bead strings to 100 helps pupils to develop their understanding of multiplication as repeated addition. Pupils should be encouraged to count in multiples as they make their groups e.g. 4 x 6 "4, 8,12,16,20,24". The colour of beads supports pupils in seeing how many groups of 10 they have to calculate more efficiently.

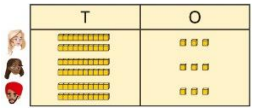
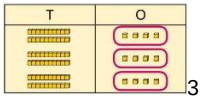
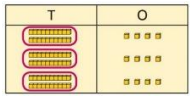
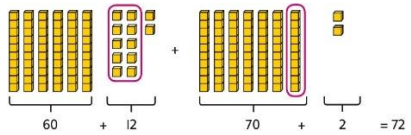
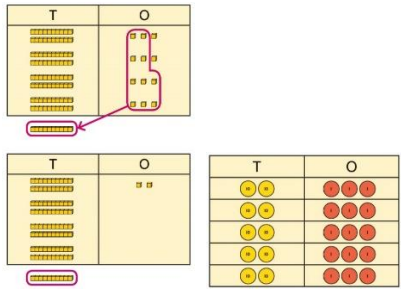
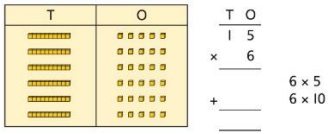
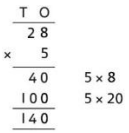
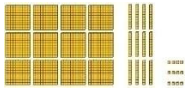
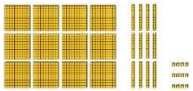
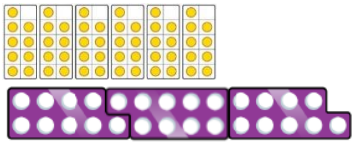
Number Tracks and Number Lines – these are useful to support pupils to count forwards in multiples. Moving counters (translucent ones are best) along the number track can support pupils to keep a track of their counting. When multiplying, children place their counter on 0 to start and then count on to find the product of numbers. Number lines and number tracks are useful with smaller multiples, however they are less efficient as numbers become larger due to the required size of the number line.

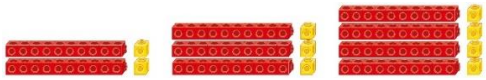
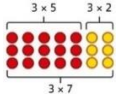
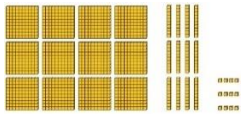
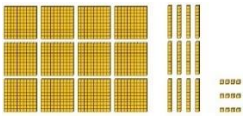
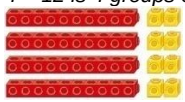
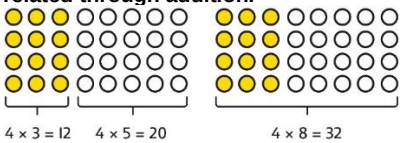
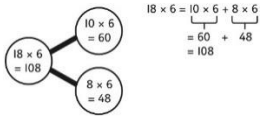
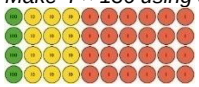
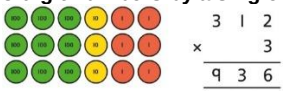
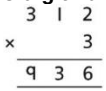
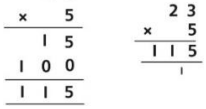
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Bar models – pupils use bar models to represent multiplications as repeated addition. They could use counters, cubes or dots within the model to support calculations before moving onto placing digits in the bars to represent the multiplication.

Base ten and place value counters – this helps to support pupils' understanding of column multiplication. It is important that pupils write out the calculation alongside the equipment. As numbers become larger, base 10 becomes less efficient. Place value counters also support the area model of multiplication as pupils can see how to multiply 2 digit numbers by 2 digit numbers.

Year 3	Concrete	Pictorial	Abstract
Understanding equal grouping and repeated addition	**Children continue to build understanding of equal groups and the relationship with repeated addition.** They recognise both examples and non-examples using objects.  Children recognise that arrays can be used to model commutative multiplications.  *I can see 3 groups of 8.* *I can see 8 groups of 3.*	**Children recognise that arrays demonstrate commutativity.**  *This is 3 groups of 4.* *This is 4 groups of 3.*	**Children understand the link between repeated addition and multiplication.**  *8 groups of 3 is 24.* *3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 24* *8 × 3 = 24* A bar model may represent multiplications as equal groups.  *6 × 4 = 24*
Using commutativity to support understanding of the times-tables	**Understand how to use times-tables facts flexibly.**  *There are 6 groups of 4 pens.* *There are 4 groups of 6 bread rolls.* *I can use 6 × 4 = 24 to work out both totals.*	**Understand how times-table facts relate to commutativity.**  *6 × 4 = 24* *4 × 6 = 24*	**Understand how times-table facts relate to commutativity.** *I need to work out 4 groups of 7.* *I know that 7 × 4 = 28* *so, I know that* *4 groups of 7 = 28* *and* *7 groups of 4 = 28.*
Understanding and using ×3, ×2, ×4 and ×8 tables.	**Children learn the times-tables as 'groups of', but apply their knowledge of commutativity.**  *I can use the ×3 table to work out how many keys.* *I can also use the ×3 table to work out how many batteries.*	**Children understand how the ×2, ×4 and ×8 tables are related through repeated doubling.**  3 × 2 = 6 3 × 4 = 12 3 × 8 = 24	**Children understand the relationship between related multiplication and division facts in known times-tables.**  *2 × 5 = 10 5 × 2 = 10* *10 ÷ 5 = 2 10 ÷ 2 = 5*
Using known facts to multiply 10s, for example 3 × 40	**Explore the relationship between known times-tables and multiples of 10 using place value equipment.** *Make 4 groups of 3 ones.*  *Make 4 groups of 3 tens.*  *What is the same?* *What is different?*	**Understand how unitising 10s supports multiplying by multiples of 10.**  *4 groups of 2 ones is 8 ones.* *4 groups of 2 tens is 8 tens.* *4 × 2 = 8* *4 × 20 = 80*	**Understand how to use known times-tables to multiply multiples of 10.**  *4 × 2 = 8* *4 × 20 = 80*
Multiplying a 2-digit number by a 1-digit number	**Understand how to link partitioning a 2-digit number with multiplying.** *Each person has 23 flowers.* *Each person has 2 tens and 3 ones.*	**Use place value to support how partitioning is linked with multiplying by a 2-digit number.** *3 × 24 = ?*	**Use addition to complete multiplications of 2-digit numbers by a 1-digit number.** *4 × 13 = ?*

	 *There are 3 groups of 2 tens.* *There are 3 groups of 3 ones.* Use place value equipment to model the multiplication context.  *There are 3 groups of 3 ones.* *There are 3 groups of 2 tens.*	 3 × 4 = 12  3 × 20 = 60 *60 + 12 = 72* *3 × 24 = 72*	*4 × 3 = 12* *4 × 10 = 40* *12 + 40 = 52* *4 × 13 = 52*
Multiplying a 2-digit number by a 1-digit number, expanded column method	**Use place value equipment to model how 10 ones are exchanged for a 10 in some multiplications.** *3 × 24 = ?* *3 × 20 = 60* *3 × 4 = 12*  *3 × 24 = 60 + 12* *3 × 24 = 70 + 2* *3 × 24 = 72*	**Understand that multiplications may require an exchange of 1s for 10s, and also 10s for 100s.** *4 × 23 = ?*  *4 × 23 = 92* *5 × 23 = ?* *5 × 3 = 15* *5 × 20 = 100* *5 × 23 = 115*	**Children may write calculations in expanded column form, but must understand the link with place value and exchange.** Children are encouraged to write the expanded parts of the calculation separately.  *5 × 28 = ?* 
Year 4	**Concrete**	**Pictorial**	**Abstract**
Multiplying by multiples of 10 and 100	**Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.**  *3 groups of 4 ones is 12 ones.* *3 groups of 4 tens is 12 tens.* *3 groups of 4 hundreds are 12 hundreds.*	**Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.**  *3 × 4 = 12* *3 × 40 = 120* *3 × 400 = 1,200*	**Use known facts and understanding of place value and commutativity to multiply mentally.** *4 × 7 = 28* *4 × 70 = 280* *40 × 7 = 280* *4 × 700 = 2,800* *400 × 7 = 2,800*
Understanding times-tables up to 12 × 12	**Understand the special cases of multiplying by 1 and 0.**  *5 × 1 = 5* *5 × 0 = 0*	**Represent the relationship between the ×9 table and the ×10 table.** 	**Understand how times-tables relate to counting patterns.** Understand links between the ×3 table, ×6 table and ×9 table *5 × 6 is double 5 × 3* ×5 table and ×6 table

		Represent the ×11 table and ×12 tables in relation to the ×10 table.  *2 × 11 = 20 + 2* *3 × 11 = 30 + 3* *4 × 11 = 40 + 4*  *4 × 12 = 40 + 8*	*I know that 7 × 5 = 35* *so I know that 7 × 6 = 35 + 7.* ×5 table and ×7 table *3 × 7 = 3 × 5 + 3 × 2*  ×9 table and ×10 table *6 × 10 = 60* *6 × 9 = 60 − 6*
Multiplying by multiples of 10 and 100	**Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.**  *3 groups of 4 ones is 12 ones.* *3 groups of 4 tens is 12 tens.* *3 groups of 4 hundreds are 12 hundreds.*	**Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.**  *3 × 4 = 12* *3 × 40 = 120* *3 × 400 = 1,200*	**Use known facts and understanding of place value and commutativity to multiply mentally.** *4 × 7 = 28* *4 × 70 = 280* *40 × 7 = 280* *4 × 700 = 2,800* *400 × 7 = 2,800*
Understanding and using partitioning in multiplication	**Make multiplications by partitioning.** *4 × 12 is 4 groups of 10 and 4 groups of 2.*  *4 × 12 = 40 + 8*	**Understand how multiplication and partitioning are related through addition.**  *4 × 3 = 12* *4 × 5 = 20* *12 + 20 = 32* *4 × 8 = 32*	**Use partitioning to multiply 2-digit numbers by a single digit.** *18 × 6 = ?*  *18 × 6 = 10 × 6 + 8 × 6* *= 60 + 48* *= 108*
Column multiplication for 2- and 3-digit numbers multiplied by a single digit	**Use place value equipment to make multiplications.** *Make 4 × 136 using equipment.*  *I can work out how many 1s, 10s and 100s.* *There are 4 × 6 ones… 24 ones* *There are 4 × 3 tens … 12 tens* *There are 4 × 1 hundreds … 4 hundreds* *24 + 120 + 400 = 544*	**Use place value equipment alongside a column method for multiplication of up to 3-digit numbers by a single digit.** 	**Use the formal column method for up to 3-digit numbers multiplied by a single digit.**  Understand how the expanded column method is related to the formal column method and understand how any exchanges are related to place value at each stage of the calculation. 

UKS2

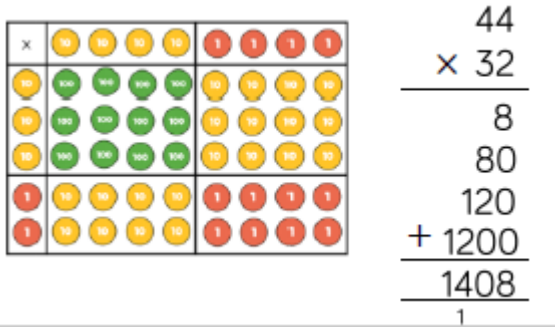
Progression within Multiplication

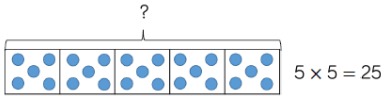
Multiplication Skills: Building on their understanding, pupils develop methods to multiply up to 4-digit numbers by single-digit and 2-digit numbers.
Pupils develop column methods with an understanding of place value, and they continue to use the key skill of unitising to multiply by 10, 100 and 1,000.
Multiplication of decimals is introduced and refined in Year 6.

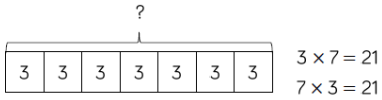
Key vocabulary: decimal, column methods, exchange, partition, mental method, ten thousand, hundred thousand, million, factor, multiple, prime number, square number, cube number, multiple, multiple, multiplier, multiplicand, product.

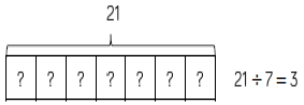
Pupils should be taught the correct vocabulary for the structure of a multiplication equation: factor x factor = product

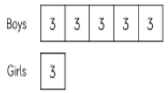
Resources and Representations







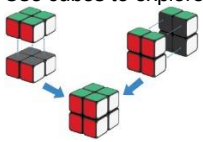
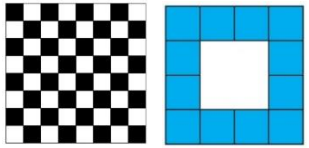
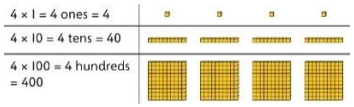
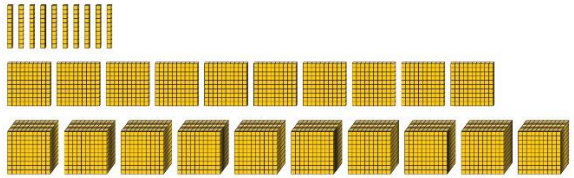
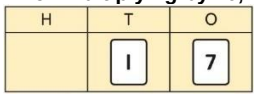
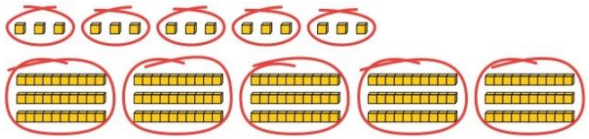
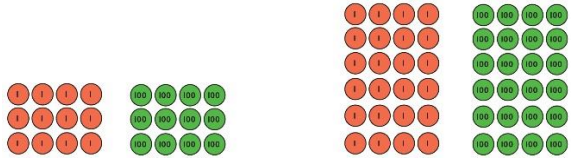
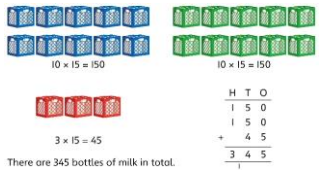
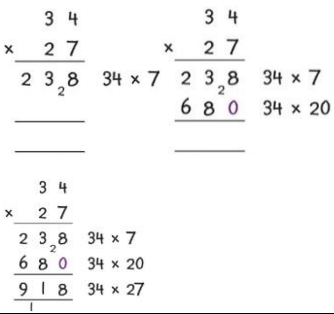


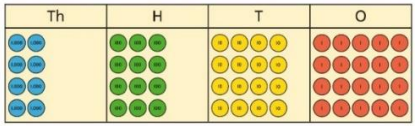
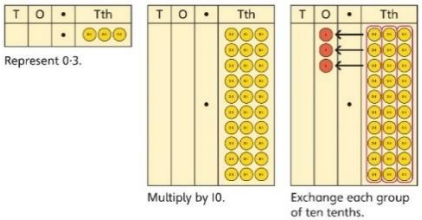
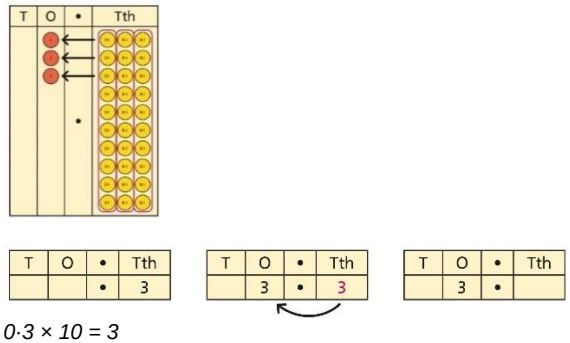


Why are these representations important?

Bar models – pupils use bar models to represent multiplications as repeated addition. They could use counters, cubes or dots within the model to support calculations before moving onto placing digits in the bars to represent the multiplication. Bar models are useful for scaling problems e.g there are 3 girls in a group, there are 5 times more boys than girls, how many boys are there?

Place value counters – they help to support pupils' understanding of column multiplication. It is important that pupils write out the calculation alongside the equipment. Place value counters also support the area model of multiplication as pupils can see how to multiply 2 digit numbers by 2 digit numbers.

Year 5	Concrete	Pictorial	Abstract
Understanding factors	**Use cubes or counters to explore the meaning of 'square numbers.** *25 is a square number because it is made from 5 rows of 5.* Use cubes to explore cube numbers.  *8 is a cube number.*	**Use images to explore examples and non-examples of square numbers.**  *8 × 8 = 64* *8^2 = 64* *12 is not a square number, because you cannot multiply a whole number by itself to make 12.*	**Understand the pattern of square numbers in the multiplication tables.** Use a multiplication grid to circle each square number. Can children spot a pattern?
Multiplying by 10, 100 and 1,000	**Use place value equipment to multiply by 10, 100 and 1,000 by unitising.** 	**Understand the effect of repeated multiplication by 10.** 	**Understand how exchange relates to the digits when multiplying by 10, 100 and 1,000.**  *17 × 10 = 170* *17 × 100 = 17 × 10 × 10 = 1,700* *17 × 1,000 = 17 × 10 × 10 × 10 = 17,000*
Multiplying by multiples of 10, 100 and 1,000	Use place value equipment to explore multiplying by unitising.  *5 groups of 3 ones is 15 ones.* *5 groups of 3 tens is 15 tens.* *So, I know that 5 groups of 3 thousands would be 15 thousands.*	Use place value equipment to represent how to multiply by multiples of 10, 100 and 1,000.  *4 × 3 = 12* *4 × 300 = 1,200* *6 × 4 = 24* *6 × 400 = 2,400*	Use known facts and unitising to multiply. *5 × 4 = 20* *5 × 40 = 200* *5 × 400 = 2,000* *5 × 4,000 − 20,000* *5,000 × 4 = 20,000*
Multiplying 2-digit numbers by 2-digit numbers	**Partition one number into 10s and 1s, then add the parts.** *23 × 15 = ?*  *23 × 15 = 345*		**Use column multiplication, ensuring understanding of place value at each stage.** 

Multiplying up to 4-digits by 2-digits			**Use column multiplication, ensuring understanding of place value at each stage.** 1 4 3 × 1 2 2 8 6 — 143 × 2 1 4 3 0 — 143 × 10 1 7 1 6 — 143 × 12 Progress to include examples that require multiple exchanges as understanding, confidence and fluency build. *1,274 × 32 = ?* *First multiply 1,274 by 2.* 1 2 7 4 × 3 2 2 5 4 8 — 1,274 × 2 1 2 7 4 × 3 2 2 5 4 8 — 1,274 × 2 3 8 2 2 0 — 1,274 × 30 *Then multiply 1,274 by 30. Finally, find the total.* 1 2 7 4 × 3 2 2 5 4 8 — 1,274 × 2 3 8 2 2 0 — 1,274 × 30 4 0 7 6 8 — 1,274 × 32 *1,274 × 32 = 40,768*
Year 6	**Concrete**	**Pictorial**	**Abstract**
Multiplying up to a 4-digit number by a 2-digit number	**Use equipment to explore multiplications.**  *4 groups of 2,345* *This is a multiplication:* *4 × 2,345* *2,345 × 4*		**Use compact column multiplication with understanding of place value at all stages.** 1 2 3 5 × 2 1 1 2 3 5 — 1 × 1,235 2 4 7 0 0 — 20 × 1,235 2 5 9 3 5 — 21 × 1,235
Multiplying by 10, 100 and 1,000	**Use place value equipment to explore exchange in decimal multiplication.**  *0·3 × 10 = ?* *0·3 is 3 tenths.* *10 × 3 tenths are 30 tenths.* *30 tenths are equivalent to 3 ones.*	**Understand how the exchange affects decimal numbers on a place value grid.**  *0·3 × 10 = 3*	**Use knowledge of multiplying by 10, 100 and 1,000 to multiply by multiples of 10, 100 and 1,000.** *8 × 100 = 800* *8 × 300 = 800 × 3* *= 2,400* *2·5 × 10 = 25* *2·5 × 20 = 2·5 × 10 × 2* *= 50*

Progression within division

Reception

Reception pupils focus on concrete and pictorial representations. At this stage, pupils focus on representing objects in different ways e.g. understanding that 5 cars can also be represented as 5 counters, 5 cubes, 5 pictures of cars, etc.

Pupils develop the core ideas that underpin all calculation. They begin by connecting calculation with counting on, developing an understanding of wholes and parts to enable them to calculate efficiently and accurately, and with greater flexibility. Pupils record their calculations in their own ways, **there is no expectation of number sentences at this stage**, however pupils may choose this way to record their thinking.

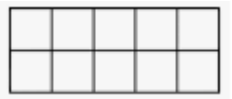
Pupils first start to look at the idea of equal groups through their exploration of doubles. They use five frames and objects to check that groups are equal.

Pupils then explore halving numbers by making two equal groups. They highlight patterns between doubling and halving seeing that double 2 is 4 and half of 4 is 2.

Pupils explore sharing into more than two equal groups. They share objects one by one, ensuring that each group has an equal share.

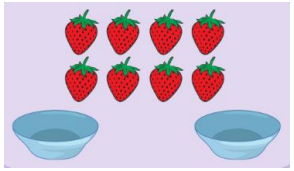
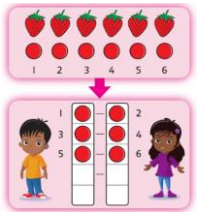
Key Vocabulary: count, , whole, part, recombine, group, share, equal groups, divide, shared equally





Why are these representations important?

Counters and concrete objects – pupils need lots of experience of putting objects into equal groups, exploring division as sharing. Five frames and tens frames help pupils to see that the groups they are making are equal.

Division	Real-life representation	Other representations
Halving and sharing	 Children explore halving and sharing through practical sharing using real life scenarios including sharing fruit or classroom equipment. Half of 8 is 4	 Children use five frames to share amounts fairly and to check that the groups are equal. They share the counters/cubes one by one. *Half of 6 is 3.*

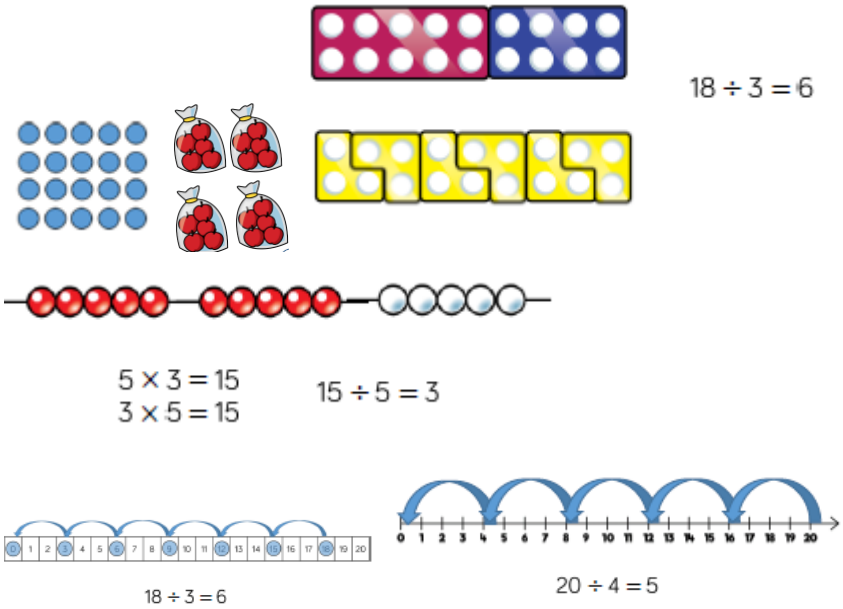
Key Stage 1

Progression within division

Pupils develop an awareness of equal groups and link this with counting in equal steps, starting with 2s, 5s and 10s.
In Year 2, they learn to connect the language of equal groups with the mathematical symbols for multiplication and division.
They learn how division can be related to repeated subtraction to find the answer to the calculation.
Pupils explore and experience a variety of images and manipulative representations of equal groups, including concrete experiences as well as abstract calculations.

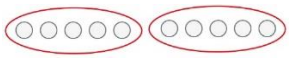
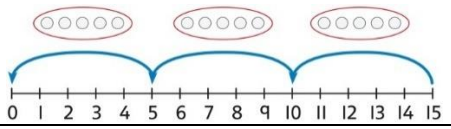
Key language: whole, part, ones, ten, group, share, equal, is equal to, groups, equal groups, divide, share, shared equally, times-table, divisor, dividend, quotient.
Pupils should be taught the correct vocabulary for the structure of a division equation: dividend ÷ divisor = quotient

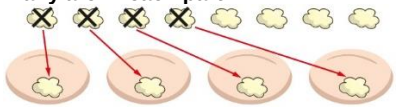
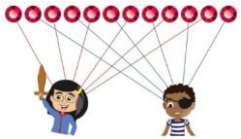
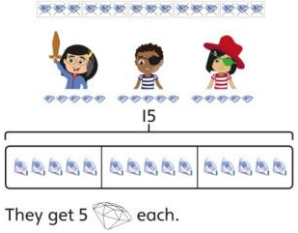
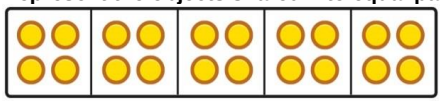
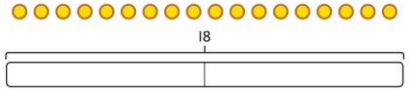
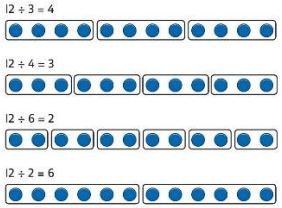
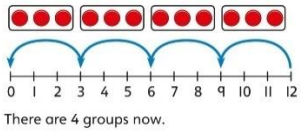
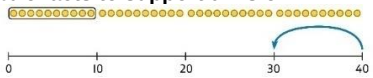
Resources and Representations

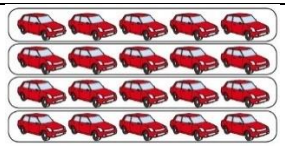
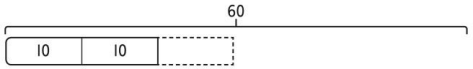
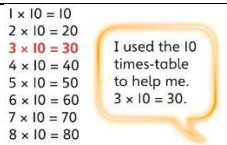


Why are these representations important?

Concrete objects and counters – pupils need lots of experience of using practical equipment to develop understanding of division and **sharing** and as **grouping.**
Sharing begins with the number of groups and the total number of objects you want to share between them. The aim is to find out how many objects each (equal) group can receive e.g. We have 12 biscuits and we want to share them among three friends, how many biscuits does everyone get?
When **grouping**, the number of objects each group should receive is known as well as the total number of objects you can distribute. The aim is to find out how many (equal) groups can be created.
e.g. We want to make boxes of biscuits for our friends. Each box will have 4 biscuits. We have 12 biscuits in total. How many boxes can we make?
Numicon – when dividing Numicon helps to develop pupils' understanding of division as grouping. Pupils make the number they are dividing(the dividend) and then place the divisor on top of the number to find how many groups of the division there are.
Arrays – these hep pupils to develop an understanding of the relationship between multiplication and division.
Bead strings – these are best used to develop an understanding of grouping. When dividing, pupils build the number they are dividing and then group the beads in the number they are dividing by (the divisor), they then count how many groups they have .
Number track and number lines – when dividing, pupils place their counter on the number they are dividing and count back in jumps of the divisor, recording how many jumps they have made.

Year 1	Concrete	Pictorial	Abstract
	Grouping **Learn to make equal groups from a whole and find how many equal groups of a certain size can be made.** Sort a whole set people and objects into equal groups. 	**Grouping** **Represent a whole and work out how many equal groups.**  *There are 10 in total.* *There are 5 in each group.*	**Grouping** **Children may relate this to counting back in steps of 2, 5 or 10.** 

	There are 10 children altogether. *There are 2 in each group.* *There are 5 groups.*	*There are 2 groups.*	
	Sharing **Share a set of objects into equal parts and work out how many are in each part.** 	**Sharing** **Sketch or draw to represent sharing into equal parts. This may be related to fractions.** 	**Sharing** ***10 shared into 2 equal groups gives 5 in each group.***
Year 2	**Concrete**	**Pictorial**	**Abstract**
Sharing equally	**Start with a whole and share into equal parts, one at a time.**  *12 shared equally between 2.* *They get 6 each.* Start to understand how this also relates to grouping. To share equally between 3 people, take a group of 3 and give 1 to each person. Keep going until all the objects have been shared  *15 shared equally between 3.* *They get 5 each.*	**Represent the objects shared into equal parts using a bar model.**  *20 shared into 5 equal parts.* *There are 4 in each part.*	**Use a bar model to support understanding of the division.**  *18 ÷ 2 = 9*
Grouping equally	**Understand how to make equal groups from a whole.**  *8 divided into 4 equal groups.* *There are 2 in each group.*	**Understand the relationship between grouping and the division statements.** 	**Understand how to relate division by grouping to repeated subtraction.**  *12 divided into groups of 3.* *12 ÷ 3 = 4* *There are 4 groups.*
Using known times-tables to solve divisions	**Understand the relationship between multiplication facts and division.**	**Link equal grouping with repeated subtraction and known times-table facts to support division.**  *40 divided by 4 is 10.*	**Relate times-table knowledge directly to division.**

	 *4 groups of 5 cars is 20 cars in total.* *20 divided by 4 is 5.*	Use a bar model to support understanding of the link between times-table knowledge and division. 	 *I know that 3 groups of 10 makes 30, so I know that 30 divided by 10 is 3.* *3 × 10 = 30 so 30 ÷ 10 = 3*

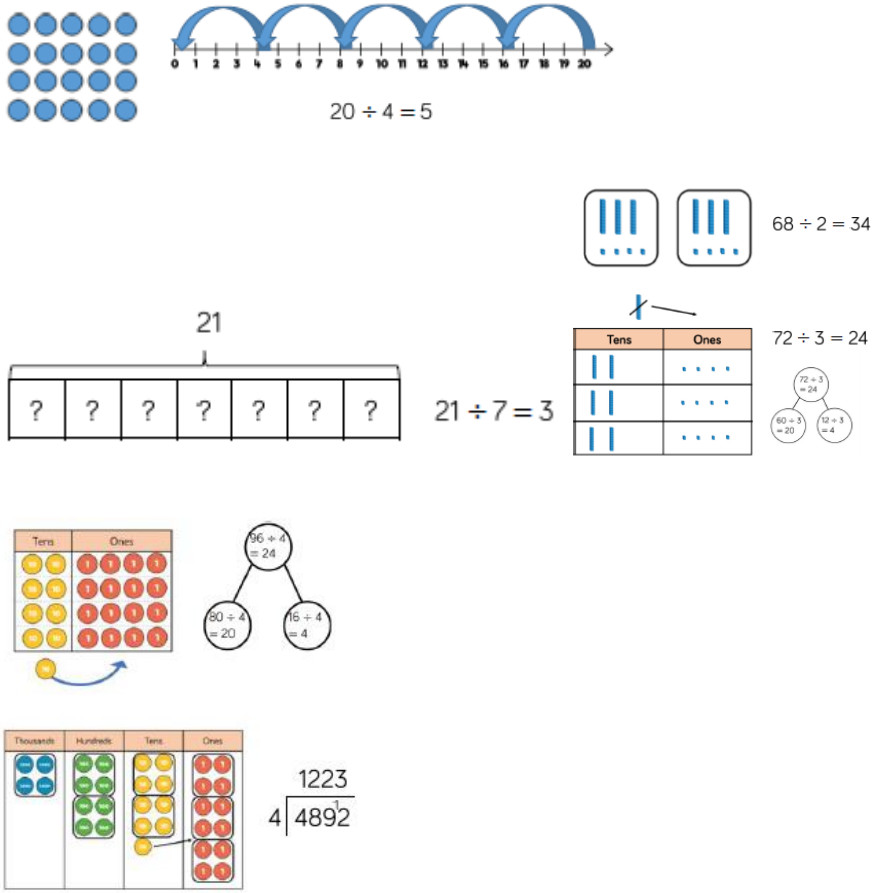
LKS2

Progression within division

Pupils build a solid grounding in times-tables, understanding the multiplication and division facts in tandem. As such, they should be as confident knowing that 35 divided by 7 is 5 as knowing that 5 times 7 is 35.
Unitising allows pupils to use known facts to multiply divide multiples of 10 and 100 efficiently.
An understanding of partitioning allows pupils to extend their skills to dividing 2- and 3-digit numbers by a single digit.
For successful division, pupils will need to make choices about how to partition. For example, to divide 423 by 3, it is effective to partition 423 into 300, 120 and 3, as these can be divided by 3 using known facts.
Pupils will also need to understand the concept of remainder, in terms of a given calculation and in terms of the context of the problem.

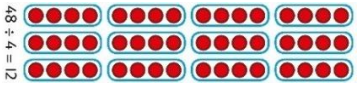
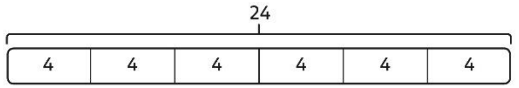
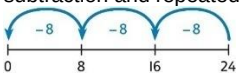
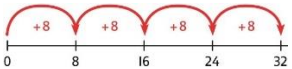
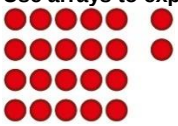
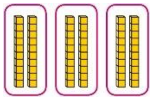
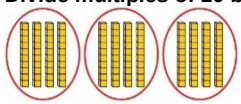
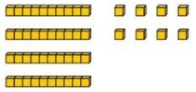
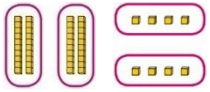
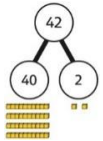
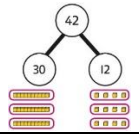
Key language: partition, place value, tens, hundreds, thousands, whole, part, equal groups, sharing, grouping, dividend, divisor, quotient, bar model, remainder, array.
Pupils should be taught the correct vocabulary for the structure of a division equation: dividend ÷ divisor = quotient

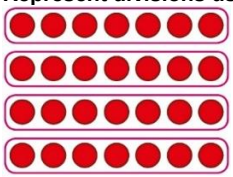
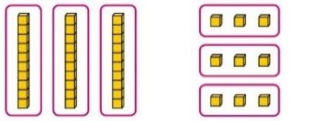
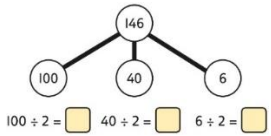
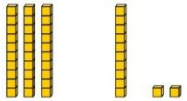
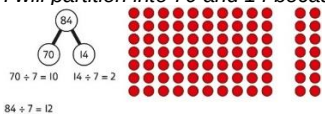
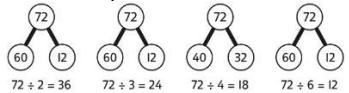
Resources and Representations

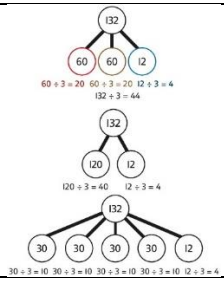
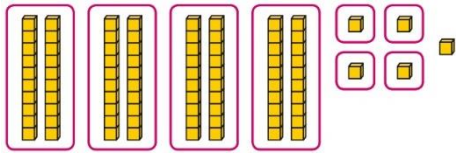
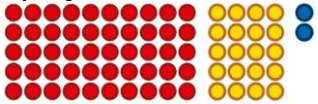
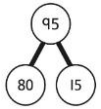


Why are these representations important?

Arrays – these hep pupils to develop an understanding of the relationship between multiplication and division.
Number track and number lines – when dividing, pupils place their counter on the number they are dividing and count back in jumps of the divisor, recording how many jumps they have made.
Bar Models – division can be represented by showing the total of the bar model and then dividing the part model into equal parts, each part representing the divisor.
Base 10 – when sharing, pupils start with the larger place value and work from left to right. If there are any left in a column, they exchange the tens for ten ones. When recording, pupils should be encouraged to use a part whole model to consider how the number has been partitioned in order to divide. This will also support them with mental methods.
Place value counters – these support pupils' understanding of short division by grouping counters rather than sharing them. Pupils work form left to right through the place value columns, grouping them according to the divisor. If there are any counters left over after they have been grouped, they can exchange the counter e.g. exchange 100 for ten tens.

Year 3	Concrete	Pictorial	Abstract
Using times-tables knowledge to divide	**Use knowledge of known times-tables to calculate divisions.**  *24 divided into groups of 8.* *There are 3 groups of 8.*	**Use knowledge of known times-tables to calculate divisions.**  *48 divided into groups of 4.* *There are 12 groups.* *4 × 12 = 48* *48 ÷ 4 = 12*	**Use knowledge of known times-tables to calculate divisions.** *I need to work out 30 shared between 5.* *I know that 6 × 5 = 30* *so I know that 30 ÷ 5 = 6.* A bar model may represent the relationship between sharing and grouping.  *24 ÷ 4 = 6* *24 ÷ 6 = 4* Children understand how division is related to both repeated subtraction and repeated addition.  *24 ÷ 8 = 3*  *32 ÷ 8 = 4*
Understanding remainders	**Use equipment to understand that a remainder occurs when a set of objects cannot be divided equally any further.**  *There are 13 sticks in total.* *There are 3 groups of 4, with 1 remainder.*	**Use arrays to explain remainders.**  *22 ÷ 5 = 4 remainder 2*	**Understand that the remainder is what cannot be shared equally from a set.** *22 ÷ 5 = ?* *3 × 5 = 15* *4 × 5 = 20* *5 × 5 = 25 … this is larger than 22* *So, 22 ÷ 5 = 4 remainder 2*
Using known facts to divide multiples of 10	**Use place value equipment to understand how to divide by unitising.** *Make 6 ones divided by 3.*  *Now make 6 tens divided by 3.*  *What is the same? What is different?*	**Divide multiples of 10 by unitising.**  *12 tens shared into 3 equal groups.* *4 tens in each group.*	**Divide multiples of 10 by a single digit using known times-tables.** *180 ÷ 3 = ?* *180 is 18 tens.* *18 divided by 3 is 6.* *18 tens divided by 3 is 6 tens.* *18 ÷ 3 = 6* *180 ÷ 3 = 60*
2-digit number divided by 1-digit number, no remainders	**Children explore dividing 2-digit numbers by using place value equipment.**  *48 ÷ 2 = ?* *First divide the 10s.*  *Then divide the 1s.*	**Children explore which partitions support particular divisions.**  *I need to partition 42 differently to divide by 3.* 	**Children partition a number into 10s and 1s to divide where appropriate.**  *60 ÷ 2 = 30* *8 ÷ 2 = 4* *30 + 4 = 34* *68 ÷ 2 = 34* Children partition flexibly to divide where appropriate. *42 ÷ 3 = ?* *42 = 40 + 2* *I need to partition 42 differently to divide by 3.* *42 = 30 + 12*

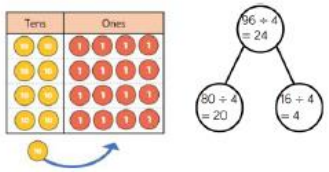
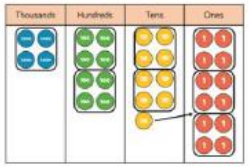
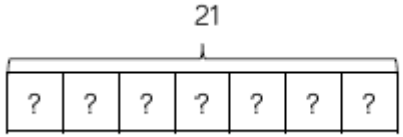
		42 = 30 + 12 *42 ÷ 3 = 14*	*30 ÷ 3 = 10* *12 ÷ 3 = 4* *10 + 4 = 14* *42 ÷ 3 = 14*
2-digit number divided by 1-digit number, with remainders	**Use place value equipment to understand the concept of remainder.** *Make 29 from place value equipment.* *Share it into 2 equal groups.*  *There are two groups of 14 and 1 remainder.*	**Use place value equipment to understand the concept of remainder in division.** *29 ÷ 2 = ?*  *29 ÷ 2 = 14 remainder 1*	**Partition to divide, understanding the remainder in context.** *67 children try to make 5 equal lines.* *67 = 50 + 17* *50 ÷ 5 = 10* *17 ÷ 5 = 3 remainder 2* *67 ÷ 5 = 13 remainder 2* *There are 13 children in each line and 2 children left out.*
Year 4	**Concrete**	**Pictorial**	**Abstract**
Understanding the relationship between multiplication and division, including times-tables	**Use objects to explore families of multiplication and division facts.**  *4 × 6 = 24* *24 is 6 groups of 4.* *24 is 4 groups of 6.* *24 divided by 6 is 4.* *24 divided by 4 is 6.*	**Represent divisions using an array.**  28 ÷ 7 = 4	**Understand families of related multiplication and division facts.** *I know that 5 × 7 = 35* *so, I know all these facts:* *5 × 7 = 35* *7 × 5 = 35* *35 = 5 × 7* *35 = 7 × 5* *35 ÷ 5 = 7* *35 ÷ 7 = 5* *7 = 35 ÷ 5* *5 = 35 ÷ 7*
Dividing 2-digit and 3-digit numbers by a single digit by partitioning into 100s, 10s and 1s	**Partition into 10s and 1s to divide where appropriate.** *39 ÷ 3 = ?*  *39 = 30 + 9* *30 ÷ 3 = 10* *9 ÷ 3 = 3* *39 ÷ 3 = 13*	**Partition into 100s, 10s and 1s using Base 10 equipment to divide where appropriate.** *39 ÷ 3 = ?*  3 groups of I ten 3 groups of 3 ones *39 = 30 + 9* *30 ÷ 3 = 10* *9 ÷ 3 = 3* *39 ÷ 3 = 13*	**Partition into 100s, 10s and 1s using a part-whole model to divide where appropriate.** *142 ÷ 2 = ?*  *100 ÷ 2 = 50* *40 ÷ 2 = 20* *6 ÷ 2 = 3* *50 + 20 + 3 = 73* *142 ÷ 2 = 73*
Dividing 2-digit and 3-digit numbers by a single digit, using flexible partitioning	Use place value equipment to explore why different partitions are needed. *42 ÷ 3 = ?* *I will split it into 30 and 12, so that I can divide by 3 more easily.* 	Represent how to partition flexibly where needed. *84 ÷ 7 = ?* *I will partition into 70 and 14 because I am dividing by 7.*  84 ÷ 7 = 12	Make decisions about appropriate partitioning based on the division required.  Understand that different partitions can be used to complete the same division.

			
Understanding remainders	**Use place value equipment to find remainders.** *85 shared into 4 equal groups* *There are 24, and 1 that cannot be shared.* 	**Represent the remainder as the part that cannot be shared equally.**  *72 ÷ 5 = 14 remainder 2*	**Understand how partitioning can reveal remainders of divisions.**  *80 ÷ 4 = 20* *12 ÷ 4 = 3* *95 ÷ 4 = 23 remainder 3*

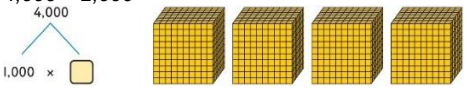
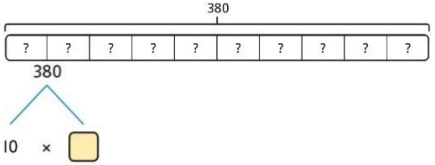
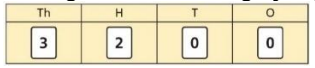
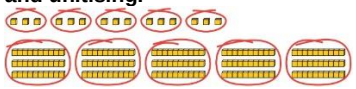
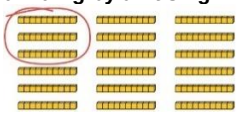
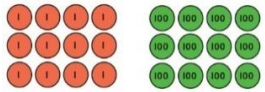
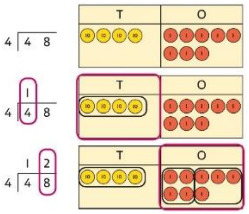
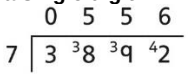
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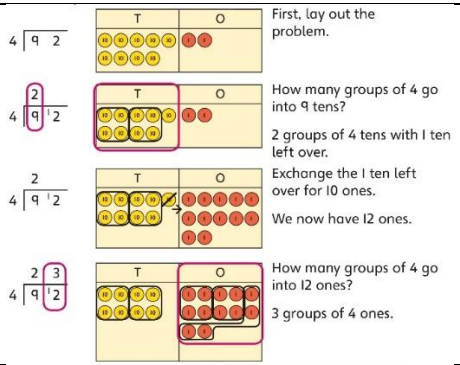
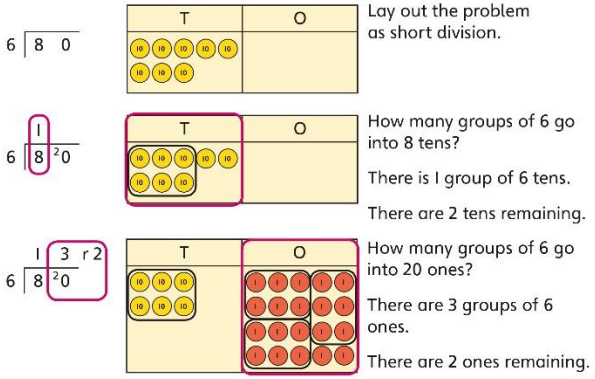
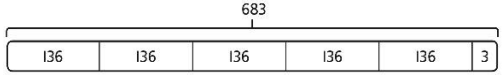
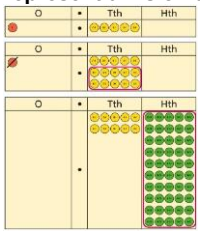
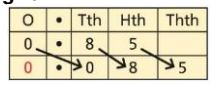
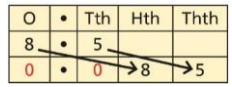
Progression within division

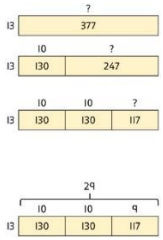
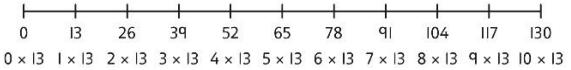
Pupils continue to use the key skill of unitising to divide by 10, 100 and 1,000. Written division methods are introduced and adapted for division by single-digit and 2-digit numbers and are understood alongside the area model and place value. In Year 6, pupils develop a secure understanding of how division is related to fractions. Multiplication and division of decimals are also introduced and refined in Year 6.
Key language: partition, place value, tens, hundreds, thousands, whole, part, equal groups, sharing, grouping, dividend, divisor, quotient, bar model, remainder, factor, prime number, composite number **Pupils should be taught the correct vocabulary for the structure of a division equation: dividend ÷ divisor = quotient**

Resources and Representations	**Why are these representations important?**
  1223 4 | 4892  21 ÷ 7 = 3	**Bar models** – division can be represented by showing the total of the bar model and then dividing the part model into equal parts, each part representing the divisor. **Place value counters** – these support pupils' understanding of short division by grouping counters rather than sharing them. Pupils work form left to right through the place value columns, grouping them according to the divisor. If there are any counters left over after they have been grouped, they can exchange the counter e.g. exchange 100 for ten tens.

Year 5	Concrete	Pictorial	Abstract
Understanding factors and prime numbers	**Use equipment to explore the factors of a given number.**  *24 ÷ 3 = 8* *24 ÷ 8 = 3* *8 and 3 are factors of 24 because they divide 24 exactly.* 24 ÷ 5 = 4 remainder 4.  *5 is not a factor of 24 because there is a remainder.*	**Understand that prime numbers are numbers with exactly two factors.** *13 ÷ 1 = 13* *13 ÷ 2 = 6 r 1* *13 ÷ 4 = 4 r 1* *1 and 13 are the only factors of 13.* *13 is a prime number.*	**Understand how to recognise prime and composite numbers.** *I know that 31 is a prime number because it can be divided by only 1 and itself without leaving a remainder.* *I know that 33 is not a prime number as it can be divided by 1, 3, 11 and 33.* *I know that 1 is not a prime number, as it has only 1 factor.*

Dividing whole numbers by 10, 100 and 1,000	**Use place value equipment to support unitising for division.** *4,000 ÷ 1,000*  *4,000 is 4 thousands.* *4 × 1,000= 4,000* *So, 4,000 ÷ 1,000 = 4*	**Use a bar model to support dividing by unitising.** *380 ÷ 10 = 38*  *380 is 38 tens.* *38 × 10 = 380* *10 × 38 = 380* *So, 380 ÷ 10 = 38*	**Understand how and why the digits change on a place value grid when dividing by 10, 100 or 1,000.**  *3,200 ÷ 100 = ?* *3,200 is 3 thousands and 2 hundreds.* *200 ÷ 100 = 2* *3,000 ÷ 100 = 30* *3,200 ÷ 100 = 32* *So, the digits will move two places to the right.*
Dividing by multiples of 10, 100 and 1,000	**Use place value equipment to represent known facts and unitising.**  *15 ones put into groups of 3 ones. There are 5 groups.* *15 ÷ 3 = 5* *15 tens put into groups of 3 tens. There are 5 groups.* *150 ÷ 30 = 5*	**Represent related facts with place value equipment when dividing by unitising.**  *180 is 18 tens.* *18 tens divided into groups of 3 tens. There are 6 groups.* *180 ÷ 30 = 6*  *12 ones divided into groups of 4. There are 3 groups.* *12 hundreds divided into groups of 4 hundreds. There are 3 groups.* *1200 ÷ 400 = 3*	**Reason from known facts, based on understanding of unitising. Use knowledge of the inverse relationship to check.** *3,000 ÷ 5 = 600* *3,000 ÷ 50 = 60* *3,000 ÷ 500 = 6* *5 × 600 = 3,000* *50 × 60 = 3,000* *500 × 6 = 3,000*
Dividing up to four digits by a single digit using short division	**Explore grouping using place value equipment**. *268 ÷ 2 = ?* *There is 1 group of 2 hundreds.* *There are 3 groups of 2 tens.* *There are 4 groups of 2 ones.* *264 ÷ 2 = 134*	**Use place value equipment on a place value grid alongside short division.**  Lay out the problem as a short division. *There is 1 group of 4 in 4 tens.* *There are 2 groups of 4 in 8 ones.* Work with divisions that require exchange.	**Use short division for up to 4-digit numbers divided by a single digit.**  *3,892 ÷ 7 = 556* Use multiplication to check. *556 × 7 = ?* *6 × 7 = 42* *50 × 7 = 350* *500 × 7 = 3500* *3,500 + 350 + 42 = 3,892*

			
Understanding remainders	**Understand remainders using concrete versions of a problem.** *80 cakes divided into trays of 6.*  *80 cakes in total. They make 13 groups of 6, with 2 remaining.*	**Use short division and understand remainders as the last remaining 1s.** 	**In problem solving contexts, represent divisions including remainders with a bar model.**  *683 = 136 × 5 + 3* *683 ÷ 5 = 136 r 3*
Dividing decimals by 10, 100 and 1,000	**Understand division by 10 using exchange.** *2 ones are 20 tenths.* *20 tenths divided by 10 is 2 tenths.*	**Represent division using exchange on a place value grid.**  *1·5 is 1 one and 5 tenths.* *This is equivalent to 10 tenths and 50 hundredths.* *10 tenths divided by 10 is 1 tenth.* *50 hundredths divided by 10 is 5 hundredths.* *1·5 divided by 10 is 1 tenth and 5 hundredths.* *1·5 ÷ 10 = 0.15*	**Understand the movement of digits on a place value grid.**  *0·85 ÷ 10 = 0·085*  *8·5 ÷ 100 = 0·085*
Year 6	**Concrete**	**Pictorial**	**Abstract**
Dividing by a 2-digit number using factors	**Understand that division by factors can be used when dividing by a number that is not prime.**	**Use factors and repeated division.** *1,260 ÷ 14 = ?*  *1,260 ÷ 2 = 630* *630 ÷ 7 = 90*	**Use factors and repeated division where appropriate.** *2,100 ÷ 12 = ?*

		1,260 ÷ 14 = 90	2,100 → ÷ 2 → ÷ 6 → 2,100 → ÷ 6 → ÷ 2 → 2,100 → ÷ 3 → ÷ 4 → 2,100 → ÷ 4 → ÷ 3 → 2,100 → ÷ 3 → ÷ 2 → ÷ 2 →
Dividing by a 2-digit number using long division	**Use equipment to build numbers from groups**.  *182 divided into groups of 13.* *There are 14 groups.*	**Use an area model alongside written division to model the process.** *377 ÷ 13 = ?*  *377 ÷ 13 = 29*	**Use long division where factors are not useful (for example, when dividing by a 2-digit prime number).** **Write the required multiples to support the division process.** *377 ÷ 13 = ?*  13 | 3 7 7 − 1 3 0 (10) 2 4 7 − 1 3 0 (10) 1 1 7 − 1 1 7 (9) 0 (29) *377 ÷ 13 = 29* A slightly different layout may be used, with the division completed above rather than at the side. 3 21 | 7 9 8 − 6 3 0 1 6 8 3 8 21 | 7 9 8 − 6 3 0 1 6 8 − 1 6 8 0